# Simulation of a supersonic plasma jet with recombination in a magnetic nozzle

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At the present time, an advanced open-ended mirror-like plasma propulsions are considered for human mission to Mars [1]. Thrust up to  $10^3$  H is needed at the stage of acceleration and braking, which can be attained at respectively small velocity of the ion or atom outflow up to  $u \ge 10^4$  m/s at the power of the order of 10 MW. Plasma jet of high density transforming into atomic flow is actual for the high thrust mode. In the present paper transformation of supersonic jet of dense plasma flowing in the divergent magnetic field into atomic flow is considered as a result of recombination at triple collisions.

Behavior of electron temperature and plasma recombination rate in the magnetic nozzle was analytically estimated disregarding the electron heating at recombination in [2] using threeparticle recombination rate depending on density *n* and electron temperature *T* as  $\alpha \propto n \cdot T^{-9/2}$ [3]. This dependence is correct for very low electron temperatures only. In fact, the electron temperature in the magnetic nozzle may be higher, and the dependence  $\alpha = \alpha(n,T)$  becomes more complicated. In the present paper numerical calculations were carried out for electron temperature and recombination rate according to  $\alpha = \alpha(n,T)$  [4] subject to electron heating at recombination.

In the cold plasma, the radiation impact recombination is the most fast recombination process in those triple collisions that show the electron capture not at the main level but at the upper atomic levels. The capture to the upper level is only the initial stage of recombination followed by a complex deactivation process. Impact transitions between the neighboring levels prevail in the area of large quantum numbers, and the further transition to the main state is realized mostly by quantum radiation. Dielectric recombination is not considered to play an important role in the further discussion.

#### Model

Due to high plasma density the ion and electron free path length is lower than the characteristic system dimensions, and study of plasma jet flow is fulfilled in the context of two-liquid magnetic hydrodynamics [5, 6]. State of every charged component is described by macroscopic parameters of density n, temperature T and flow velocity u. The plasma is fully ionized and contains one sort of ions. Effective charge of the plasma is considered to be equal to unit and the longitudinal current is not considered to be present. Viscosity and thermo-conductivity are taken into account. Earlier, we have already tried to solve the similar problem of plasma parameters determination at the axis regardless of recombination [7]. In that paper the model, boundary conditions, difference scheme, stability and convergence condition are described in detail. In the present paper, the same code is used completed with the radiation impact recombination influence at triple collisions.

Every recombination act is supposed to exclude an ion and an electron out of consideration, and every recombination act is supposed to result in heating electrons by  $\mathcal{E}$  value. The heating is related to both the presence of an electron as a third particle in the process of recombination and the deactivation of the electron due to pair collisions. Radiation arising at the atom deactivation appears to effect plasma heating negligibly and it is taken out of consideration. We failed to find any data on the electron heating value at recombination. We

assume  $\mathcal{E} = 2 \text{ eV}$  at our calculations. Experimental data on the radiation shock recombination rate in triple collisions are also very few. The results of calculations on hydrogen are the most precise data, they are shown in tables in [4]. Coefficient  $\alpha$  is described accurate within 20 % in the density range of  $10^{13} \div 10^{16} \text{ cm}^{-3}$  and temperature range of  $0.2 \div 5 \text{ eV}$  by the following equation:

$$\alpha = 10^{-13} \left\{ \frac{5.517}{T_e} + 10 \cdot T_e^{-4.1 + 1.3 \cdot T_e - 0.17 \cdot T_e^2} \right\} \cdot \left( \frac{n}{10^{14}} \right)^{0.55 + 0.03 \left( T_e - 2.7 \right)^2 + 0.006 \cdot T_e^2 + \frac{0.04}{T_e}}.$$

Boundary conditions in the throat (z = 0) were set as constants, and in the output as free heat and mass transition.

#### Simulation results and discussion

Several variants of plasma flow in a magnetic nozzle are presented that differ in longitudinal distribution of magnetic field and boundary conditions in the critical section (in the throat).

At one-dimensional adiabatic supersonic flow of ideal gas in Laval nozzle the flow velocity in the critical section is equal to transsonic speed  $c_s = \sqrt{\gamma RT}$ , and at the high enough expansion of asymptotic of gas flow with the adiabat index  $\gamma = 5/3$  are the following: the flow velocity tends to the double velocity in the critical section, the density decreases linearly as S section increases, and temperature decreases as  $T \sim S^{-2/3}$  [8].

Peak flow rate in the throat for plasma is  $c = \sqrt{\gamma k(T_i + T_e)/M}$ , where  $\gamma = 5/3$  for simple plasma. At further adiabatic plasma flow, the flow rate increase is  $\sqrt{2}$  times higher than gas flow rate increase. The additional acceleration is due the electric field that arises in the dilator to provide quasineutrality and decelerates the more mobile electrons and accelerates the ions flowing out. The electric field not only keeps electrons but cools them unlike collisionless plasma. The additional acceleration mechanism is also connected to heat arrival as a result of high longitudinal electron heat conductivity.

In rough estimation, assuming the three-particle recombination rate to be  $\alpha \propto n \cdot T^{-9/2}$ , we get the following dependence of the mutual neutralization of the charged particles on section:

$$\frac{d N_{\pm}}{d t} = S \Delta z \frac{d n_{\pm}}{d t} = S \Delta z \alpha n^2 \propto S \Delta z n^3 T_e^{-4.5} \propto S \Delta z$$

It is obvious that the most efficient transformation of a plasma flow into atom flow takes place in the range of high enough expansion of supersonic plasma flow. Heating at recombination changes this simple dependence, so more precise calculations are required.

In Fig.1, the calculation results for hydrogen plasma flow in a magnetic field of a single coil 90 cm diameter are shown both ignoring recombination and taking it into account. It is clear that recombination results in electron heating and flow velocity increase. In the following variant considered, the magnetic field of the coil was completed with the area of constant magnetic field and the results are shown in Fig.2. Recombination results in substantial temperature increase in the homogeneous area and to sizeable decrease in plasma flow transformation into atom flow from 12 % (estimated without taking into account the temperature increase) up to 4 %. To transform efficiently the plasma jet into the atomic one, it is necessary to find the way to cool electrons.

The first way is to provide a decrease of magnetic field in the recombination area in contrast to the constant one in Fig.2. Divergence is to provide the main part of atomic energy to concentrate in the longitudinal velocity. Due to plasma density decrease resulting from

expansion and recombination, the electric field occurs cooling electrons. In Fig.3, the calculation results are shown for the jet flow in the magnetic field appropriate for the electron cooling. Higher values of electron temperature in the recombination area (compared to the one in Fig.2) should not mislead as in this case the flow of higher density is considered, and energy emission due to recombination depends on density as  $n^2$ . Following this way allows to transform 27 % of plasma jet into the atomic one at the length considered.

Another way of cooling electrons is using cold gas injected into the recombination area. As the substance expenditure increases, a light gas (e.g., hydrogen) is permitted to be introduced into the plasma jet together with heavy ions (e.g., Kr). Electrons are cooled due to the molecule excitation, ionization and dissociation processes. Light atom cold ions which appear as a result of recharge will also contribute into the electron cooling. The detailed consideration of this way of cooling of electrons is not the matter of this paper.

The calculations show a significant distinction of electric field potential from the potential defined by Boltsmann's relation  $e\varphi - e\varphi_0 = kT_e \ln(n/n_0)$ . It was cleared out that the main difference is connected with the longitudinal electron thermoforce influence [9], resulting from electron temperature gradient.

Let us note for the conclusion that for lack of experimental data on recombination sections and electron heating value during the recombination act, experiments are desired for the further consideration of the problem. Comparison of experimental and calculations will allow to avoid vagueness related to the recombination section and heating value.

### Conclusion

In the present paper, dense supersonic plasma jet flowing in the magnetic nozzle and its transformation into the neutral gas flow due to the radiation impact recombination in triple collisions was analyzed for high thrust mode of plasma engine. A computer modeling of plasma flowing subject to recombination was fulfilled in the context of two-liquid magnetic hydrodynamics. Two ways of necessary electron cooling were proposed, by electric field due to expansion, and light gas injection. To determine electric field potential, the thermoforce was shown to be necessarily taken into account.

- [1] Chang-Diaz F. R. Fusion Technology 35 n. 1T (1999) 87.
- [2] Cohen S. A., Yaeyoung Park, Malkin V., et. al. Bulletin of APS 42 (1997) 1974.
- [3] Rayzer Yu. P. Gas discharge physics. M.: Nauka, 1987.
- [4] Smirnov B. M. Atomic collisions and elementary processes in plasma. Atomizdat, 1968.
- [5] Braginskii S. I. *Reviews of Plasma Physics*, Leontovich M. A., New York: Consultants Bureau, 1965, vol. 1, p. 205.
- [6] Dawson J. M., and Uman F. M. Nuclear Fusion 5 (1965) 242.
- [7] Kabantsev A. A., Sokolov V. G., Taskaev S. Yu. Plasma Phys. Reports 21 (1995) 735.
- [8] Loitsyansky L. G. Fluid and gas mechanics. M.: Nauka, 1978.
- [9] Taskaev S. Yu. Plasma Phys. Reports 23 (1997) 1042.

![](_page_3_Figure_1.jpeg)

Fig.1. Longitudinal distributions of magnetic field B, temperature T, density n, and flow velocity u and supersonic plasma jet in the magnetic nozzle ignoring recombination (dotted line), subject to recombination (full line). Boundary conditions in the throat (z = 0):  $n = 10^{16} \text{ cm}^{-3}$ , T = 3 eV.

![](_page_3_Figure_3.jpeg)

Fig.2. Longitudinal distributions of magnetic field B, temperature T, density n, and flow velocity u and supersonic plasma jet in the magnetic nozzle ignoring recombination (dotted line), subject to recombination (full line). Boundary conditions in the throat (z = 0):  $n = 1.5 \times 10^{15} \text{ cm}^{-3}$ , T = 3 eV.

![](_page_3_Figure_5.jpeg)

Fig.3. Longitudinal distributions of magnetic field B, temperature T, density n, and flow velocity u and supersonic plasma jet in the magnetic nozzle ignoring recombination (dotted line), subject to recombination (full line). Boundary conditions in the throat (z = 0):  $n = 10^{16} \text{ cm}^{-3}$ , T = 3 eV.