ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СОАН СССР

ПРЕПРИНТ И Я Ф 76 - 69

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Новосибирск
1976

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Abstract

Under some laboratory and astrophysical conditions the magnetic flux in a plasma may be concentrated in narrow, mutually far-removed tubes ("magnetic filaments"). Long-wave sound vibrations of such systems are studied on basis of the equations of magnetic hydrodynamics. It is demonstrated that even in absence of dissipative processes (viscosity, thermal conduction, Ohmic losses) the sound vibrations are absorbed as a result of an effect which in certain respects is similar to Landau damping effect and which consists in resonance excitation of bending waves propagating along the magnetic filaments. The contribution to the damping due to scattering of the sound waves by the filaments is found. Conditions under which scattering is insignificant are indicated. Damping of a low (but finite)-amplitude monochromatic sound wave is considered.

1. Introduction

In the present paper, we obtain the equations describing the propagation of waves in the plasma containing the system of "magnetic filaments" (magnetic flux tubes). The structures of such a kind apparently exist in some Solar chromospheric regions /1,2/. They can also appear near the boundary between the plasma and magnetic field when the flute instability develops.

We suppose that the radius a of filaments is small in comparison with the average distance l between them (small concentration of filaments). At the same time, we assume that the characteristic dimensions of the problem are sufficiently large for the single-fluid magnetohydrodynamics to be valid. The magnetic field outside the tubes is considered to be small. These assumptions are, apparently, adequate for some regions of Solar chromosphere.

As mentioned in Ref. /3/, such a system has an interesting feature: the long wave ($\lambda \gg l$) acoustic oscillations in the system are damped even in the absence of dissipative effects (viscosity, thermal conductivity, Ohmic losses) due to the following effect similar to the Landau damping. Namely, the bending oscillations can propagate along a separate filament with the phase velocity $H/[4\pi(\rho_e+\rho_i)]^{1/2}$, where H is the magnetic field inside and ρ_i and ρ_e are the densities inside and outside of the filament, respectively. If the plane acoustic wave propagates in the plasma and the angle θ between the direction of propagation and the direction of filaments satisfies the condition

$$\cos\theta = V_s/u \quad , \tag{1}$$

where V_s is the sound velocity in plasma, then the resonant transfer of the wave energy to the energy of filament oscillations takes place. Since in general the density and the magnetic field strength inside the different tubes are different, so the velocity u changes from one filament to another, and hence at each propagation angle θ there are filaments for which the condition (1)*) is satisfied and which absorb the energy of sound wave.

The scheme of solving this problem is as follows. First, we consider the motion of a separate filament with respect to fluid and find the interaction force between the filament and the fluid. Then, by the averaging over the volume containing many filaments (but still small in comparison with the wave-length), we obtain the expression for volume force by which the filaments act on the fluid and which enters the macroscopic equations of motion of fluid. After that we study the dispersion characteristics of the system and find out the decrement of the sound wave.

2. Equations of motion

Let us take the initial direction of filaments along the z-axis. The displacement of filament from equilibrium position is characterized by the vector $\vec{\xi}(z,t)$ perpendicular to the z-axis. Since relative velocity of the fluid and filaments is the quantity of the first order of magnitude we can neglect the change of dimension and cross section form of filament in calculating interaction force of the fluid and filaments and consider the cross section of filament to be a circle of radius a. Taking into account all mentioned above we can write down the following equation of bending oscillations of filament

$$\rho_i \pi a^2 \frac{\partial^2 \vec{\xi}}{\partial t^2} = \rho_e \pi a^2 \frac{\partial \vec{V}_\perp}{\partial t} + \rho_e \pi a^2 \left(\frac{\partial \vec{V}_\perp}{\partial t} - \frac{\partial^2 \vec{\xi}}{\partial t^2} \right) + \frac{H^2}{4\pi} \pi a^2 \frac{\partial^2 \vec{\xi}}{\partial z^2} \quad (2)$$

Let us explain the meaning of each term in right-hand side of this equation: $\vec{V}_\perp$ is the perpendicular component of macroscopic fluid velocity; the first term discribes the force by which accelerating fluid acts on the tube placed inside it; the second term is due to effect of adding mass**) (see, for example

*) This condition is analogous to the Cherenkov resonance condition in the theory of Landau damping.

**) It is easy to show, that adding mass per unit length of the tube is equal to $\pi a^2 \rho_e$.

/4/); finally the last term is the returning force with which the magnetic field confined inside the filament acts on the filament. The equation (2) does not contain the longitudinal component of the fluid velocity because longitudinal motion of the fluid does not result in the interaction between the fluid and filaments.

Speaking of macroscopic velocity of the fluid one has to bear in mind that the fluid velocity in the vicinity of some filament (at the distance of the order of a) differs considerably from the macroscopic velocity. This fact, generally speaking, should be taken into account in calculations of the latter. However, the concentration of filaments being small, in the first order approximation with respect to concentration we can regard the macroscopic velocity as the fluid velocity at the distances from filaments large as compared to a but small as compared to l . That is what we have done in deriving equation (2).

Using the concept of adding mass we assume that the fluid is incompressible. It is true since the velocities $\partial\vec{\xi}/\partial t$ and $\vec{V}_\perp$ are small as compared to sound velocity and the period of long-wave oscillations under consideration is large with respect to the eigenperiod of radial oscillations of filament.

Each filament is characterized by the radius a , plasma density inside the filament ρ_i (below we use the dimensionless parameter $\eta = \rho_i/\rho_e$ instead of ρ_i), and the temperature of this plasma T_i . Generally speaking, these parameters change from one filament to another.

For simplicity we assume that the plasma inside the filaments is cold, $T_i \ll T_e$, and, respectively, we neglect the gas-kinetic pressure p_i inside the filaments (this assumption is not principal but allows us to make all formulas not so long). Then, in an unperturbed state,

$$H^2/8\pi = p_e \equiv \rho_e V_s^2/\gamma$$

where p_e is the pressure outside the filament and γ is the specific heats ratio. Thus, the equation (2) can be written as

$$\frac{\partial^2 \vec{\xi}_\eta}{\partial t^2} - \frac{2}{\gamma}\frac{v_s^2}{1+\eta}\frac{\partial^2 \vec{\xi}_\eta}{\partial z^2} = \frac{2}{1+\eta}\frac{\partial \vec{v}_\perp}{\partial t}.$$

Here and below we mark the displacement $\vec{\xi}$ by index η to show the dependence of displacement on η.

We define the distribution function of filaments over the parameters a and η by the following formula:

$$d\alpha = f(a,\eta)\, da\, d\eta$$

where $d\alpha$ is the fraction of volume occupied with filaments with parameters a and η lying in the intervals $(a, a+da)$, $(\eta, \eta+d\eta)$. The normalization of the function f thus defined is apparently as follows:

$$\alpha = \int_0^\infty \int_0^\infty da\, d\eta\, f(a,\eta)$$

Here α is the total fraction of volume occupied with filaments ($\alpha \sim a^2/l^2 \ll 1$). It is also useful to introduce the function

$$g(\eta) = \int_0^\infty f(a,\eta)\, da \qquad (4)$$

One can see from (2) that the force with which the filament acts on the fluid (per unit length of filament) is

$$-\pi a^2 \rho_e \left(2\frac{\partial \vec{v}_\perp}{\partial t} - \frac{\partial^2 \vec{\xi}_\eta}{\partial t^2}\right)$$

Accordingly we obtain for the force acting by the filaments on the unit volume of fluid

$$\vec{F} = -\rho_e \int \left(2\frac{\partial \vec{v}_\perp}{\partial t} - \frac{\partial^2 \vec{\xi}_\eta}{\partial t^2}\right) g(\eta)\, d\eta \qquad (5)$$

Now, the macroscopic equation of motion of fluid can be represented as:

$$\rho_e \frac{\partial \vec{v}}{\partial t} = -v_s^2 \nabla \delta\rho + \vec{F} \qquad (6)$$

where $\delta\rho$ is the disturbance of fluid density which is connected with $\vec{v}$ by the continuity equation:

$$\frac{\partial}{\partial t}\delta\rho + \rho_e \operatorname{div}\vec{v} = 0 \qquad (7)$$

In writing down the continuity equation (7) and the first term in the right-hand side of equation (6), we restrict ourselves by the zeroth order approximation with respect to parameter α, because the account of the first (and higher) order terms leads only to an insignificant change of sound velocity. On the contrary, the force $\vec{F}$ (which is also of the order of α) in the equation (6) results in the qualitatively new effects connected with the bending elasticity of filaments.

Different from the known problem of oscillations of the fluid containing gas bubbles (see, for instance, Ref. /5/), filaments compressibility is inessential in our system. The reason is that the compressibilities of filaments and of surrounding medium are of the same order of magnitude in our system.

3. Damping of sound waves

The equations (3) and (5) - (7) constitute the closed set of equations describing the linear oscillations of medium. Let us consider the travelling wave type eigensolutions of this system, i.e. let us assume that $\vec{\xi}_\eta$, $\vec{v}$ and $\delta\rho_e$ are proportional to $\exp(-i\omega t + i\vec{k}\vec{r})$. Expressing $\vec{\xi}_\eta$ in terms of $\vec{v}_\perp$ from equation (2),

$$\vec{\xi}_\eta = \frac{2i\omega\vec{v}_\perp}{\omega^2(1+\eta) - 2\gamma^{-1} v_s^2 k^2 \cos^2\theta} \qquad (8)$$

and substituting the result in equation (5), we obtain

$$\vec{F} = i\omega\rho_e \vec{v}_\perp I(\omega,\vec{k}) \qquad (9)$$

$$I(\omega,\vec{k})=2\int_0^\infty d\eta\, g(\eta)\left[1-\left(\eta+1-\frac{2v_s^2k^2\cos^2\theta}{(\omega+i0)^2}\right)^{-1}\right]$$

We take into account that disturbance must vanish at $t\to-\infty$ and accordingly we replace ω by $\omega+i0$ in denominator of the integrand.

The influence of magnetic filaments on the oscillations enters the problem by the integral I which is of the order of α and consequently is much smaller than unity. Taking this into account it is easy to get the dispersion relation from equations (6), (7) and (9):

$$\omega\simeq kv_s\left[1-\frac{\sin^2\theta}{2}I(\omega,\vec{k})\right]$$

(for definiteness, we consider the solution with $Re\,\omega>0$).

In calculating the integral I we can put the frequency to be equal to the solution of dispersion relation for "pure" fluid (i.e. the fluid without filaments): $\omega=kv_s$. Further it should be noted that the real part of I leads only to an insignificant correction to the oscillation's frequency, so that it is sufficient to calculate the imaginary part of I. Using the well known equality:

$$Im\frac{1}{x+i0}=-i\pi\delta(x),$$

it is easy to get the following expression for damping rate $\nu\equiv-Im\,\omega$:

$$\nu=\frac{kv_s\sin^2\theta}{2}Im\,I=\pi kv_s\sin^2\theta\begin{cases}g(\eta_0), & \eta_0>0,\\ 0, & \eta_0<0,\end{cases}\qquad(10)$$

$$\eta_0=\frac{2\cos^2\theta}{\gamma}-1$$

Of course, we could get the same result in a more formal way by solving the Cauchy problem for the system (3) - (7) by means of Laplace transform (in the same way as it is done by Landau /6/ in the initial-value problem for Langmuir oscillations).

If we make a natural assumption that the width $\Delta\eta$ of the region of those values of η, where the distribution function is essentially nonzero, is of the order of unity*), then $g(\eta_0)\sim\alpha$, and for damping rate we can write down the following estimate:

$$\nu/kv_s\sim\alpha\qquad(11)$$

Of course, this estimate holds only for those values of θ, where $\eta_0>0$, i.e. where $\cos\theta>(\gamma/2)^{1/2}$. For monatomic gas the corresponding region is quite narrow, $\theta\lesssim\arccos(5/6)^{1/2}\simeq5^\circ$, so that it seems to be essential to take into account the possible noncollinearity of separate filaments. Just this problem is treated in what follows in this section.

The direction of a separate filament can be characterized by the unit vector $\vec{n}$ directed along the filament's axis. Let us denote by $h(\vec{n})$ the distribution function of filaments over the directions. At the same time we assume for simplicity that all filaments have the same radius and the same density. Normalization of function $h(\vec{n})$ is the following:

$$\alpha=\int h(\vec{n})do$$

where α is the part of a volume occupied by filaments and do is the element of a solid angle. The component of macroscopic velocity perpendicular to vector $\vec{n}$ is obviously equal to $\vec{v}-\vec{n}(\vec{n}\vec{v})$. Accordingly, instead of (8) we have:

$$\xi_{\vec{n}}=\frac{2i\omega[\vec{v}-\vec{n}(\vec{n}\vec{v})]}{\omega^2(1+\eta)-2\gamma^{-1}v_s^2(\vec{k}\vec{n})^2}$$

In this case, since displacement apparently depends on filament's orientation, we mark ξ by subscript $\vec{n}$ (as to parameter η, it is assumed here to be fixed). By analogy with (9), the volume force acting on plasma, can be presented in the following way:

$$F_\alpha=i\omega\rho_e K_{\alpha\beta}v_\beta\,,$$

where

*) That is, we assume that the substance density inside the filaments changes with respect to ρ_e not more than by several times from one filament to another.

$$K_{\alpha\beta}=2\int do\, h(\vec{n})(\delta_{\alpha\beta}-n_\alpha n_\beta)\left[1-\left(1+\eta-\frac{2v_s^2(\vec{k}\vec{n})^2}{\gamma(\omega+i0)^2}\right)^{-1}\right]$$

Having this expression for the force and using equations (6) and (7) it easy to write down the dispersion relation. Then, taking into account the smallness of $K_{\alpha\beta}$ we can find the small imaginary part of frequency:

$$\frac{\nu}{kv_s}=\frac{k_\alpha k_\beta}{2k^2}\,\mathrm{Im}\,K_{\alpha\beta}=\int do\, h(\vec{n})\left[1-\frac{(\vec{k}\vec{n})^2}{k^2}\right]\delta\left[1+\eta-\frac{2(\vec{k}\vec{n})^2}{\gamma k^2}\right]$$

In an important particular case when distribution of filaments is isotropic we have $h(\vec{n})=\alpha/4\pi$ and the expression for damping rate is reduced to simple form:

$$\frac{\nu}{kv_s}=\frac{\sqrt{2}}{8}\,\frac{\alpha\gamma\left[2-\gamma(1+\eta)\right]}{\sqrt{\gamma(1+\eta)}}$$

(we imply that $\eta<2/\gamma-1$).

Up to now, considering the equation of motion of a separate filament (see equation (2)) we completely neglected the compressibility of medium. It is justified by the smallness of the oscillation's frequency $\omega=kv_s$ in comparison with v_s/a . Account for the compressibility (that is account for the higher order terms with respect to parameter $\omega a/v_s\sim ka$) results in a new effect: radiation of acoustic waves at bending oscillations of filaments. The radiative damping of bending oscillations arising from this effect has a damping rate $\nu_{rad}\sim\omega k^2a^2$ (see Appendix).

The previous consideration is valid only if damping rate of acoustic waves ν is greater than ν_{rad} , that is if

$$(ka)^2<\alpha \qquad (12)$$

If this condition is satisfied, the energy of acoustic waves is transferred into the energy of filament's oscillations during the time of the order of ν^{-1}. Then, for a much greater time of the order of ν_{rad}^{-1} filaments release their energy in the form of secondary acoustic waves. By noting that the separation l between filaments is of the order of $a/\sqrt{\alpha}$ one can write

down inequality (12) in the form: $kl\ll 1$. In other words, if macroscopic description is valid ($k\ll 1/l$), then inequality (12) is satisfied automatically.

One should note that, for calculating of small damping rate of acoustic wave, the macroscopic approach is generally speaking not necessary. It is sufficient to consider the excitation of separate filaments and then to use the energy conservation law for finding the damping rate of initial wave. However, the above approach has the advantage as it better permits to show the analogy with Landau damping effect.

4. The damping due to the scattering by filaments

In this section we elucidate the connection between the Landau damping considered above and the damping connected with the scattering of acoustic waves by filaments.

As it is shown in Appendix, the energy Q of secondary waves radiated from the unit length of filament per unit time is connected with the energy density of initial acoustic wave as follows:

$$Q=\beta(\eta,a,\omega)W \qquad (13)$$

where

$$\beta=\beta_0+2\sum_{m=1}^{\infty}\beta_m, \qquad (14)$$

$$\beta_0=\frac{V_s}{k}\frac{\pi^2}{16}(ka)^4\left(\sin^2\theta-\frac{\gamma}{2}\right),$$

$$\beta_m=\frac{V_s}{k}\left[\frac{\pi}{m!(m-1)!(1+\eta)}\right]^2\left(\frac{ka\sin\theta}{2}\right)^{4m}\frac{\Omega^2}{(\omega-\Omega)^2+\nu_{rad}^{(m)2}}$$

$$\Omega=k_zv_s\sqrt{2/\gamma(1+\eta)},\quad \cos\theta=k_z/k$$

and $\nu_{rad}^{(m)}$ is defined by the formula (A.9).

For defineteness, we consider that all the filaments are collinear. In this case one can express the damping rate of primary acoustic wave due to scattering in terms of the coefficient β and distribution function $f(a,\eta)$ in the following way:

$$\nu_{scatt} = \int \frac{f(a,\eta)}{\pi a^2} \beta(\eta, a, \omega) d\eta d\omega \qquad (15)$$

The main contribution to damping is due to the dipole term ($m=1$) in β : the next multipoles ($m = 2, 3, \ldots$) contain the higher orders of small parameter ka , and in the term $m=0$, there is no resonant denominator. The resonances in scattering are due to the existence of weakly damped natural oscillations of filaments (see Appendix).

One should bear in mind that relationship (14) is obtained for a strictly monochromatic initial wave. If one deals with damped wave with damping rate ν larger than ν_{rad} (just as in the case $\alpha \gg (ka)^2$), then it's obviously impossible to have a frequency mismatch in denominators of β smaller than ν , and in calculating the contribution of scattering (15) one should take into account only the regions where $|\omega - \Omega| \gtrsim \nu$. Integration over the corresponding regions of η gives the following estimate for the damping rate due to scattering:

$$\nu_{scatt}/kv_s \sim (ka)^4/\alpha \qquad (16)$$

It follows from the comparison of formulae (11) and (16) that the scattering is negligible if condition (12) is satisfied.

Note that the damping rate of short waves ($a \ll \lambda \ll l$) is completely determined by scattering: it appears to be smaller than ν_{rad} (see below), and this means that damping is due to direct scattering of primary acoustic wave into the secondary waves without preliminary accumulation of energy in natural oscillations of filaments. In other words, for short waves one has $\nu = \nu_{scatt}$, where ν_{scatt} is defined by formula (15).

As was mentioned above, one needs to retain only a term $m=1$ in the expression for β . Since only a narrow region of η (of width $\Delta\eta \sim \nu_{rad}/\omega \ll 1$) around the resonance $\eta = \eta_0$ contributes to the integral (15), one can replace $f(a,\eta)$ by

$f(a,\eta_0)$. After this, integration over η and a can be done easily resulting in the following expression which formally coincides with (10):

$$\nu = \nu_{scatt} = \pi k v_s \sin^2\theta\, g(\eta_0), \; \eta_0 = 2\cos^2\theta/\gamma - 1 \quad (17)$$

We imply that $\eta_0 > 0$; in the opposite case the scattering becomes nonresonant and damping rate becomes much smaller: $\nu \sim \alpha (kv_s)(ka)^2$.

5. Damping of monochromatic acoustic wave

It was shown in the preceding two sections that the damping of long ($\lambda \gg l$) acoustic waves is completely determined by the effect analogous to Landau damping. The analogy becomes especially complete, if one makes nonlinear estimates related to the problem of the damping of monochromatic acoustic wave of small (but finite) amplitude*).

For defineteness, we shall speak of the case when all the filaments are parallel to z-axis. Assume, that at $t=0$ the acoustic wave with the amplitude of particles' displacement $\vec{\zeta}$ and the wave number $\vec{k}$ is excited in plasma. Let us consider the motion of a separate filament in the field of such a wave. For this, we use equation (3) presented in the form:

$$\frac{\partial^2 \vec{\xi}_\eta}{\partial t^2} + \Omega^2(\eta)\vec{\xi}_\eta = -\omega^2 \vec{\zeta}_\perp e^{-i\omega t} \qquad (18)$$

where

$$\Omega(\eta) = k_z v_s \sqrt{2/\gamma(1+\eta)}, \quad \omega = kv_s$$

(for a while, we assume that the amplitude of the sound wave does not change with time; this point is elucidated below).

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*) In the theory of Langmuir oscillations the corresponding estimates were made by Mazitov /7/ and O'Neil /8/; see also Kadomtsev's survey /9/.

Let's denote by η_0 the resonant value of η that is the value of η for which $\Omega(\eta)=\omega$. For those values of η which are sufficiently close to η_0, the amplitude of oscillations of filaments grows linearly with time and becomes very soon much larger than ζ_1 (because of this, one can neglect the initial value of ξ_η which should be taken to be of the order of ζ_1, cf. Ref. /9/). The saturation of amplitude occurs due to nonlinear frequency shift (see Ref. /10/), which we denote by $\Delta\Omega(\tilde{\xi}_\eta)$, where $\tilde{\xi}_\eta$ is an amplitude of ξ_η. The first nonvanishing term of the expansion of $\Delta\Omega$ in powers of $\tilde{\xi}_\eta$ has a form $\Delta\Omega = A\tilde{\xi}_\eta^2$ where A is of the order of ωk_z^2. For values of η which are close to η_0, the saturation level of amplitude can be found by means of a simple relationship, following from (18):

$$\tilde{\xi}_\eta \left| \frac{\partial\Omega}{\partial\eta}(\eta-\eta_0) + A\tilde{\xi}_\eta^2 \right| \sim \zeta_1 \omega \qquad (19)$$

Since $\partial\Omega/\partial\eta \sim \omega$ (we assume that $\eta_0 \sim 1$), it is clear from (19) that $\tilde{\xi}_\eta$ depends on η in the following way:

$$\tilde{\xi}_\eta \sim \begin{cases} \dfrac{\zeta_1}{|\eta-\eta_0|}; & |\eta-\eta_0| \gtrsim (\zeta_1 k_z)^{2/3}, \\ \zeta_1(\zeta_1 k_z)^{-2/3}; & |\eta-\eta_0| \lesssim (\zeta_1 k_z)^{2/3} \end{cases} \qquad (20)$$

(we have taken into account an estimate $A \sim \omega k_z^2$). Since the oscillation energy of filament related to its unit length is of the order of $\rho_e a^2 \tilde{\xi}^2 \omega^2$, using Eq. (20) we can estimate the maximum energy W^* which can be transferred to the filaments:

$$W^* \sim \rho_e \omega^2 \int g(\eta)\tilde{\xi}_\eta^2 d\eta \sim \rho_e \omega^2 \zeta_1^2 (\zeta_1 k_z)^{-2/3} g(\eta_0) \sim g(\eta_0)(\rho v_s^2/W)^{1/3} W$$

where $W \sim \rho_e \omega^2 \zeta^2$ is the energy density of the sound wave. Noting that $g(\eta_0) \sim \alpha$, one can rewrite the last relationship in the form:

$$W^*/W \sim \alpha (\rho v_s^2/W)^{1/3}. \qquad (21)$$

This estimate solves the problem of the damping of finite amplitude wave (cf. Ref. /9/): at $W/\rho v_s^2 > \alpha^3$ the maximum energy that can be transferred to the filaments due to resonant interaction of them with the sound wave is smaller than the energy of the wave i.e., in this case the wave can transfer to the filaments only a small fraction of its initial energy, and then the damping ceases. In this case the assumption of the constancy of the amplitude of the sound wave is valid.

On the contrary, at $W/\rho v_s^2 < \alpha^3$, the energy of filaments remains small even after complete absorption of the sound wave, and the damping is described by the linear theory presented in Sec. 3. Consequently, the condition when the linear approximation can be applied to the problem of damping of monochromatic sound wave with initial amplitude ζ, has the form:

$$k\zeta < \alpha^{3/2} \qquad (22)$$

We neglect the nonlinearity of the acoustic wave itself. It is known that this nonlinearity leads to distortion of the acoustic wave profile and to formation of discontinuities within the time $\tau \sim (\omega k \zeta)^{-1}$. Our nonlinear estimates are valid if is large with respect to the time of damping of the wave in the case (22) or with respect to saturation time in the opposite case.

Note that, due to radiative damping of oscillations of filaments, acoustic wave will be damped to extinction even in the case $k\zeta > \alpha^{3/2}$. However, the damping time appears to be much larger than ν^{-1}. In this case the dissipation rate of acoustic wave energy can be estimated as: $-\dot{W} \sim \nu_{rad} W^*$, where W^* is difined by the formula (21), and ν_{rad} is radiative damping rate. Consequently,

$$-\dot{W} \sim \nu_{rad} W \cdot \alpha (\rho v_s^2/W)^{1/3},$$

and we find for the damping time of an acoustic wave:

$$\tau \sim (1/\nu_{rad}\alpha)(W/\rho v_s^2)^{1/3} \gg 1/\nu_{rad}$$

We imply that $\nu_{rad} \ll \omega\Omega$ since otherwise the nonlinear effects become insignificant.

Appendix

Here we consider an exact theory of linear oscillations of a magnetic filament taking into account the final compressibility of external medium. For a plasma inside the filament, the linearized system of MHD-equations has a form:

$$\rho_i \frac{\partial \vec{v}}{\partial t} = \frac{1}{4\pi}\left[\mathrm{rot}\,\vec{h}, \vec{H}\right], \quad \frac{\partial \vec{h}}{\partial t} = \mathrm{rot}\left[\vec{v}\vec{H}\right],$$

where $\vec{v}$ and $\vec{h}$ are the perturbations of velocity and of magnetic field, respectively (we take into account the absence of gaskinetic pressure inside the filament).

We use the cylindrical coordinates (r, φ, z) and consider the solutions proportional to $\exp(-i\omega t + ik_z z + im\varphi)$ with $m = 0, \pm 1, \pm 2, \ldots$. It is easy to show that for these solutions

$$(\omega^2 - k_z^2 V_A^2)(\mathrm{rot}\,\vec{v})_z = 0.$$

Since we are not interested in the Alfven waves that can propagate inside the filaments, we obtain from this equation that $(\mathrm{rot}\,\vec{v})_z = 0$, that is,

$$v_r = -\frac{\partial \psi}{\partial r}, \quad v_\varphi = -\frac{im\psi}{r}, \qquad \text{(A.1)}$$

where ψ is some function of r . It follows from the equations of motion that $v_z = 0$. Perturbation of magnetic field can also be expressed in terms of ψ :

$$h_r = \frac{k_z H}{\omega}\frac{\partial \psi}{\partial r}, \; h_\varphi = \frac{imk_z H}{\omega r}\psi, \; h_z = \frac{i\omega H}{V_A^2}\left(1 - \frac{k_z^2 V_A^2}{\omega^2}\right)\psi \qquad \text{(A.2)}$$

From the condition $\mathrm{div}\,\vec{h} = 0$, one can obtain the following equation for ψ :

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \psi}{\partial r} + \left(\frac{\omega^2}{V_A^2} - k_z^2 - \frac{m^2}{r^2}\right)\psi = 0 \qquad \text{(A.3)}$$

In the region outside the filament, magnetic field is zero, and the linear motions in this region can be described by the equation for velocity potential χ ($\vec{v} = -\nabla\chi$):

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \chi}{\partial r} + \left(\frac{\omega^2}{V_s^2} - k_z^2 - \frac{m^2}{r^2}\right)\chi = 0 \qquad \text{(A.4)}$$

where V_s is related to V_A :

$$V_s^2 = \frac{\gamma\eta}{2} V_A^2 \quad . \qquad \text{(A.5)}$$

Perturbation of gaskinetic pressure can be expressed in terms of χ in the following way:

$$\delta p = \frac{i\omega}{\gamma}\rho_e \chi \qquad \text{(A.6)}$$

At the surface of filament ($r = a$), the boundary conditions of continuity of the normal component of velocity,

$$v_r|_i = v_r|_e ,$$

and of the normal component of the momentum flux,

$$H h_z / 4\pi = \delta p$$

are satisfied. Using the equations (A.1), (A.3), (A.5) and (A.6), one can reduce these boundary conditions to the single one:

$$\left(\eta - \frac{2}{\gamma}\frac{k_z^2 V_s^2}{\omega^2}\right)\frac{\partial \ln \chi}{\partial r}\bigg|_{r=a} = \frac{\partial \ln \psi}{\partial r}\bigg|_{r=a} \qquad \text{(A.7)}$$

Inside the filament, the solution is proportional to the Bessel function of the order m: $\psi \propto J_m(q_i r)$ where $q_i = (\omega^2/V_A^2 - k_z^2)^{1/2}$. In the outer region, the solution should have a form of a divergent wave: $\chi \propto H_m^{(1)}(q_e r)$, where $H_m^{(1)}$ is the Hankel function of a first kind, and $q_e = (\omega^2/V_s^2 - k_z^2)^{1/2}$, $\mathrm{Re}\,q_e > 0$. Because we consider oscillations with $k_z a \ll 1$, the arguments of J_m and $H_m^{(1)}$ in the boundary condition (A.7) are small as compared to unity. If one retains only first nonvanishing terms of the expansion in powers of these parameters , then one obtains the following dispersion relation:

$$\omega_m = \Omega \equiv k_z v_s \sqrt{2/\gamma(1+\eta)} \qquad (A.8)$$

$m = \pm 1, \pm 2, \pm 3, \ldots$ There is no radiative damping in this approximation. Note that there is no dependence of natural frequency on m (the bending oscillations correspond to the dipole mode $m = \pm 1$). There are no weakly damping oscillations with $m = 0$.

Generally speaking, at the interaction with plane acoustic wave, the modes with $m = \pm 2, \pm 3, \ldots$ are exited as well as the dipole mode considered in Sections 2, 3. However, their amplitudes are very small (the reason for this is that in the expansion of a plane wave over the cylindrical multipoles the amplitude of the mode with some m in the vicinity of the filament is proportional to $(ka)^{|m|}$).

If we retain the next term of expansion over ka in the boundary condition (A.7) this will result in: 1) small correction to the real part of freqency; 2) appearance of radiative damping. For brevity, we present here only the formula for radiative damping rate $\nu_{rad}^{(m)}$:

$$\frac{\nu_{rad}^{(m)}}{\Omega} = \frac{\pi}{|m|!\,(|m|-1)!\,(1+\eta)} \left(\frac{ak_z}{2}\right)^{2|m|} \left[\frac{2}{\gamma(1+\eta)} - 1\right]^{|m|} \qquad (A.9)$$

Let's consider now the scattering problem. The plane acoustic wave of unit amplitude has a form:

$$\chi = \frac{1}{2}\exp\left(-i\omega t + i\vec{k}\vec{r}\right) + \text{C.C.} \qquad (A.10)$$

where $\omega = kv_s$. In the presence of magnetic filament the solution outside the filament will be superposition of this plane wave (which can be represented in cylindrical coordinates as $\frac{1}{2}\exp(-i\omega t + ik_z z + iq_e r\cos\varphi) + \text{c.c.}$) and divergent cylindrical waves:

$$\chi = e^{-i\omega t + ik_z z}\left[\frac{1}{2}e^{-iq_e r\cos\varphi} + \sum_{m=-\infty}^{+\infty} A_m H_m^{(1)}(q_e r)e^{im\varphi}\right] + \text{c.c.}$$

Inside the filament the solution has a form:

$$\psi = e^{-i\omega t + ik_z z}\sum_{m=-\infty}^{+\infty} B_m J_m(q_i r)e^{im\varphi} + \text{c.c.}$$

using the identity

$$e^{iq_e r\cos\varphi} = \sum_{m=-\infty}^{+\infty} i^m J_m(q_e r)e^{im\varphi}$$

and writing the boundary condition (A.7) for each azimuthal harmonic m we obtain the following expression for the coefficients A_m :

$$A_m = -\frac{i^m}{2}\,\frac{\left(\eta - \frac{2}{\gamma}\frac{k_z^2 v_s^2}{\omega^2}\right)q_e J_m'(q_e a)J_m(q_i a) - q_i J_m'(q_i a)J_m(q_e a)}{\left(\eta - \frac{2}{\gamma}\frac{k_z^2 v_s^2}{\omega^2}\right)q_e H_m^{(1)\prime}(q_e a)J_m(q_i a) - q_i J_m'(q_i a)H_m^{(1)}(q_e a)}$$

Taking into account the smallness of parameter ka we can represent A_m for $m > 0$ in the form:

$$A_m = -\frac{\pi i^{m-1}}{2(1+\eta)\,m!\,(m-1)!}\left(\frac{ka\sin\theta}{2}\right)^{2m}\frac{\Omega}{\omega - \Omega + i\nu_{rad}^{(m)}}, \quad A_{-m} = A_m^*,$$

where $\Omega(\eta)$ is defined by the formula (A.8). Getting this expression we have taken into account that A_m is essentially nonzero only at $\omega \simeq \Omega$ and that is why we replace ω by Ω everywhere except the resonant denominators. In the wave zone at $q_e r \gg 1$ the following asymptotic formula is valid:

$$H_m^{(1)}(q_e r) \simeq \sqrt{\frac{2}{\pi q_e r}}\exp\left\{i\left(q_e r - \frac{m\pi}{2} - \frac{\pi}{4}\right)\right\}$$

Here, in small regions of space the divergent wave can be considered as a plane one. This allows one to find the density of radial energy flux in the m-order mode:

$$q_m = |A_m|^2 \rho v_s k/\pi r$$

The power radiated into the m-th mode from the unit length of the filament is $2\pi r\, q_m$. Taking into account that the energy density in the incident acoustic wave (A.10) is $\rho k^2/2$ we get the relationship (14) with

$$\beta_m = 4\,|A_m|^2 v_s/k \quad .$$

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Работа поступила - 3 декабря I975г.

Ответственный за выпуск - С.Г.ПОПОВ
Подписано к печати 9.УП-I976г. МН 07394
Усл. I,3 печ.л., тираж 200 экз. Бесплатно
Заказ № 69.

Отпечатано на ротапринте ИЯФ СО АН СССР