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GENERALIZED ENERGY PRINCIPLE FOR FLUTE PERTURBATIONS IN AXISYMMETRIC MIRROR MACHINES

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Abstract

The stability of plasma in the axisymmetric mirror device with respect to purely electrostatic flute perturbations is considered. It is shown that a population of hot particles with fast azimuthal drift may contribute in a favorable way to the stability of the system as a whole. The reason is that the fast drifting particles affect not a potential energy of the MHD perturbations but rather their kinetic energy, and in the case, when the effective kinetic energy is found to be negative, the stabilization takes place.

I Introduction

Axial symmetry is a very desirable property of the mirror devices both for fusion and neutron source applications. The main obstacle to be circumvented in the development of such systems, is the flute instability of axisymmetric mirrors. In recent years there appeared a number of proposals, devoted to the stabilization of the flute perturbations in the framework of axisymmetric magnetic configurations; most of them are based on the combining of the MHD unstable central cell with various types of end-cell stabilizers (see, e.g., the surveys [1]—[3]). In the present paper we concentrate ourselves just on this scheme, that includes a long solenoidal cell, conjugated with the end stabilizing anchor, intended to provide MHD stability of the system as a whole. The attractive feature of such a configuration is that it allows to exploit finite larmor radius (FLR) effects [4] for the stabilization of the flute perturbations. As is well known [4], FLR effects, being strong, stabilize all flute modes, except the one with azimuthal number $m = 1$, corresponding to the "rigid"

displacement of the plasma column (the "global" mode). Consequently, in the conditions when FLR effects dominate, the anchor has to stabilize the "global" mode only. Bearing in mind a favorable influence of FLR effects we, however, don't restrict our paper by discussion of only "global" mode stability and consider a general case of an arbitrary azimuthal mode.

The conventional approach to the stability analysis contains an assumption that the curvature-induced particle drift is slow with respect to the time-scale of the flute perturbation growth,

$$\Omega_d \ll \Gamma, \tag{1}$$

where Γ denotes a characteristic growth-rate of the flute perturbation (or a characteristic frequency in the stable case), and Ω_d is a characteristic drift frequency in the stabilizing anchor. Inequality (1) means that plasma particles in the anchor move together with the flux tube that they were initially occupying. Therefore their perturbed position can be defined by a 2-D hydrodynamic displacement vector $\boldsymbol{\xi}$, being the same for all particles, occupying in the initial state some flux tube, irrespective to their velocity and pitch angle. So, one can analyze the stability of the system in the framework of the familiar Kruskal-Oberman energy principle [5]. Just this approach is in the basis of the proposals mentioned in the surveys [1]—[3].

On the other hand, the presence of a long central cell, possessing large inertia, can result in the violation of (1). Indeed, flute growth-rate in the central cell itself (without the anchor) scales as [6]

$$\Gamma \sim v_{Ti}/\sqrt{L_c L_{tr}}, \tag{2}$$

where L_c, L_{tr} are lengths of solenoidal and transition regions respectively, and v_{Ti} is the ion thermal velocity in the central cell. It looks hardly probable

to achieve a stabilizing contribution of the anchor more than $1.5 - 2$ **times** exceeding the unstable contribution of the centfal cell. Hence, if one switches on the anchor, the characteristic frequency of the flute oscillations keeps its former value (2), just the sign of Γ^2 changes. Drift frequency around the magnetic axis in the anchor region is estimated as follows:

$$\Omega_d \sim \frac{cT_s}{eB_s L_s^2}, \tag{3}$$

B_s being the magnetic field in the stabilizer middle plane, T_s being plasma temperature in the stabilizer (to avoid unnecessary notation overloading in estimates, we suppose electron and ion temperatures in the anchor to be equal to each other). As it follows from (2),(3), the growth-rate Γ becomes really small as compared with the drift frequency when L_c is large enough.

For the latter reason, it is of interest to study the situation when the drift frequency in the stabilizing anchor exceeds substantially the frequency of MHD perturbations, i.e. when inequality inverse to (1) is satisfied:

$$\Omega_d \gg \Gamma. \tag{4}$$

To analyze the stability of such a system one cannot use Kruskal-Oberman energy principle and has to rely on its modified version that takes into account the condition (4).

The generalized energy principle accounting for the contribution of non-hydrodynamic (in the sense of (4)) plasma species has been formulated in [7], where single-particle adiabatic invariant technique has been used to calculate the perturbation of the particle energy. The energy variation, derived in [7], consists of the contributions of both perturbed plasma and magnetic energies. However, if one inserts into the expression for W, presented in [7],

the displacement vector $\boldsymbol{\xi}$ of the flute perturbation

$$\boldsymbol{\xi} = \frac{\nabla\eta \times \mathbf{B}}{B^2}, \tag{5}$$

with η constant along the field line, then one comes to a somewhat paradoxical result. Namely, it turns out that W, as given in [7], becomes identically zero [8]. The reason is that the authors of paper [7] retained in W only terms proportional to ξ^2 while, as it has been shown in [8, 9], the contribution of non-MHD particles to the energy of the perturbations has a different structure, being proportional to $\dot{\xi}^2$. Henceforth in this paper we consider just the case of purely flute perturbations (5).

The contents of the present paper is as follows. In the next Section we reproduce the derivation of the generalized energy principle [9] for the axisymmetric case. The generalization of the energy principle [9] to the intermediate frequency interval, $\Gamma/m \ll \Omega_d \leq \Gamma$, for modes with high azimuthal numbers is the subject of Section III. In Section IV we invoke the cited energy principle to illustrate the possibility of the stabilization of the flute instability due to the "negative inertia" effect. The study of the conditions, required for the "negative inertia" stabilization in the magnetic mirror, is performed in Sections V.1, V.2. Section VI is devoted to the application of the generalized energy principle to stability analysis of the neutron source device, proposed in [10]. Section VII contains conclusions.

II Generalized energy principle for axisymmetric plasma configurations

Suppose that in the anchor region there exists a population of the hot particles with drift frequencies satisfying inequality (4), while the plasma in the

central cell is cold enough to admit conventional MHD treatment. Througout the paper we use the $\beta \to 0$ approximation. We examine the stability of such a system with respect to the purely electrostatic perturbations that are characterized by the electrostatic potential φ, constant along the field line. The latter assumption, identical to the one made in [7], implies the presence of a cold plasma component which justifies the constancy of the potential along the field line and acts to provide quasineutrality condition.

In this paper we use the following coordinate frame: we mark every field line with the polar coordinates r, ψ of its intersection with some plane, perpendicular to the uniform magnetic field B_c in the solenoidal region of the central cell. Instead of r one can use the magnetic flux Φ inside the cylindrical surface of the radius r : $\Phi = \pi r^2 B_c$. A drift surface can then be described by the equation $\Phi = \Phi(\psi)$. In these coordinates potential φ of the perturbations, corresponding to the mode with azimuthal number m, is given by

$$\varphi = \tilde{\varphi}(\Phi)\cos m\psi. \tag{6}$$

Note that

$$\tilde{\varphi}(0) = 0, \tag{7}$$

since there is no singularity of the electric field on the magnetic axis.

At a given configuration of magnetic and electrostatic fields, the drift surface for a particle with a total energy ε and magnetic moment μ is determined by the constancy of the longitudinal action

$$J(\varepsilon, \mu, \Phi, \psi) = (2M)^{1/2} \int (\varepsilon - \mu B - e\varphi)^{1/2} dl, \tag{8}$$

with the integration carried out between the turning points. If the condition (4) is satisfied, then, with the electrostatic potential varying, the drift surface

adjusts itself so as to keep constant the magnetic flux inside the surface [11]. This occurs via variation of the particle energy.

To find the change W of the kinetic energy of the particles (just this quantity enters the energy principle for the perturbations with a scale-length much in excess of the Debye radius), we use the following approach. We consider some group of particles (of a total number ΔN) that in the initial state have certain values of ε and μ, and that are filling a drift surface characterized with a certain value of J. When we slowly turn on the electrostatic potential of the perturbation, the drift surface deforms and the kinetic energy of the particles changes. If we find the change of the kinetic energy ΔW for this group, then, by summation over all the groups, we find the required quantity W.

The group ΔN is drifting along the contour $\Phi(\psi)$ determined by the instantaneous configuration of the electrostatic field and the instantaneous value of ε . The number of particles from this group $d\Delta N$, occupying the section of the contour of the arc length $d\psi$, can be presented in the form $d\Delta N = \nu d\psi$, where ν is the number of particles per unit arc length. The stationarity condition $\nu\dot{\psi} = const$ yields:

$$\nu = \frac{\Delta N \Omega_d}{2\pi\dot{\psi}}, \tag{9}$$

where $\dot{\psi}$ is the angular velocity of the bounce-averaged drift motion [12]:

$$\dot{\psi} = -\frac{2\pi c}{e t_{\|}} J_\Phi, \tag{10}$$

$t_{\|} = J_\varepsilon$ is the transit time between the turning points, and Ω_d is the drift frequency,

$$\Omega_d = 2\pi \left[\int_0^{2\pi} \frac{d\psi}{\dot{\psi}} \right]^{-1}. \tag{11}$$

We use notations $J_\varepsilon \equiv \partial J/\partial\varepsilon$, $J_\Phi \equiv \partial J/\partial\Phi$, etc., for the partial derivatives. The change of the kinetic energy of these particles is, obviously,

$$\Delta W = \left[\int_0^{2\pi} \nu(\varepsilon - e\varphi) d\psi \right] \Bigg|_{\text{initial}}^{\text{final}}, \tag{12}$$

where the subscripts indicate the difference between the final and initial state. The total energy ε of particles is constant over the drift surface in the time-scale of $\Omega_d{}^{-1}$, while their kinetic energy $\varepsilon - e\varphi$ varies from one field line to another according to variation of φ . In this respect, ΔW, if divided by ΔN, represents the average (over the drift surface) change of the kinetic energy.

The condition of flux conservation inside the drift surface can be written in the form:

$$\int_0^{2\pi} \Phi(\psi) d\psi \Big|_{\text{initial}}^{\text{final}} = 0, \tag{13}$$

where $\Phi(\psi)$ is a solution of the equation

$$J(\varepsilon, \mu, \Phi, \psi)\Big|_{\text{initial}}^{\text{final}} = 0. \tag{14}$$

In principle, equations (9)-(14) allow one to find the particle kinetic energy at arbitrarily large φ . However, we will consider only the case of small φ . The quantities of the first order in φ will be denoted by the subscript "1", the second order corrections by the subscript "2", etc.

In the linear approximation, equation (14) yields:

$$(\varepsilon_1 - e\varphi) J_\varepsilon + \Phi_1 J_\Phi = 0, \tag{15}$$

wherefrom, taking into account relationships (6) and (13), we find that

$$\varepsilon_1 = 0, \qquad \Phi_1 = e\varphi \frac{J_\varepsilon}{J_\Phi}. \tag{16}$$

The next order expansion of (14) gives:

$$J_\varepsilon(\varepsilon_2 - e\Phi_1\varphi_\Phi) + J_\Phi\Phi_2 + \frac{1}{2}J_{\varepsilon\varepsilon}(e\varphi)^2 - J_{\varepsilon\Phi}\Phi_1 e\varphi + \frac{1}{2}J_{\Phi\Phi}{\Phi_1}^2 = 0. \tag{17}$$

The requirement (13), when applied to Φ_2 , yields:

$$\int_0^{2\pi} d\psi(\varepsilon_2 - e\Phi_1\varphi_\Phi) = -\int_0^{2\pi} \frac{d\psi}{J_\varepsilon}(\frac{1}{2}J_{\varepsilon\varepsilon}\varphi^2 - J_{\varepsilon\Phi}\Phi_1 e\varphi + \frac{1}{2}J_{\Phi\Phi}{\Phi_1}^2), \quad (18)$$

while from (12) we find that

$$\Delta W = \frac{\Delta N}{2\pi}\int_0^{2\pi}[\frac{\dot{\psi}_1}{\dot{\psi}}e\varphi + (\varepsilon_2 - e\Phi_1\varphi_\Phi)]d\psi. \quad (19)$$

From equation (10) one obtains that

$$\frac{\dot{\psi}_1}{\dot{\psi}} = -\frac{J_\varepsilon}{J_\Phi}e\varphi_\Phi + \frac{J_\varepsilon}{J_\Phi}\left(\frac{J_{\Phi\Phi}}{J_\Phi} - \frac{2J_{\varepsilon\Phi}}{J_\varepsilon} + \frac{J_{\varepsilon\varepsilon}J_\Phi}{J_\varepsilon^2}\right)e\varphi. \quad (20)$$

Now, using relationships (6), (16), (19) and (20), we can express ΔW in terms of $\tilde{\varphi}$:

$$\Delta W = -\frac{1}{4}e^2\Delta N\left[\frac{J_\varepsilon}{J_\Phi}\frac{\partial\tilde{\varphi}^2}{\partial\Phi} + \tilde{\varphi}^2\left(2\frac{J_{\varepsilon\Phi}}{J_\Phi} - \frac{J_{\varepsilon\varepsilon}}{J_\varepsilon} - \frac{J_\varepsilon J_{\Phi\Phi}}{J_\Phi^2}\right)\right]. \quad (21)$$

To perform the summation over the plasma particles, we introduce the distribution function $F(\varepsilon,\mu,\Phi)$, normalized according to the relationship $\Delta N = F(\varepsilon,\mu,\Phi)\Delta\varepsilon\Delta\mu\Delta\Phi$. Then, the energy W of the perturbation acquires its final form:

$$W = -\frac{1}{4}e^2\int d\varepsilon d\mu d\Phi F(\varepsilon,\mu,\Phi)\times$$
$$\times\left[\frac{J_\varepsilon}{J_\Phi}\frac{\partial\tilde{\varphi}^2}{\partial\Phi} + \tilde{\varphi}^2\left(2\frac{J_{\varepsilon\Phi}}{J_\Phi} - \frac{J_{\varepsilon\varepsilon}}{J_\varepsilon} - \frac{J_\varepsilon J_{\Phi\Phi}}{J_\Phi^2}\right)\right]. \quad (22)$$

Sometimes it's more convenient to use the expression for W in the form:

$$W = \frac{1}{4}e^2\int d\varepsilon d\mu d\Phi\tilde{\varphi}^2\times$$
$$\times\left[\frac{\partial}{\partial\Phi}\left(F\frac{J_\varepsilon}{J_\Phi}\right) - F\left(2\frac{J_{\varepsilon\Phi}}{J_\Phi} - \frac{J_{\varepsilon\varepsilon}}{J_\varepsilon} - \frac{J_\varepsilon J_{\Phi\Phi}}{J_\Phi^2}\right)\right], \quad (23)$$

which can be obtained from (22) through the integration by parts of the first term in the square brackets (22). There is no contribution from the

integration limits, since on the upper limit there are no particles, and so the distribution function equals to zero, while on the lower limit the potential φ vanishes according to (7).

In Appendix, we derive also the expression for the energy variation, relevant to the conditions when the longitudinal energy of the particles is small as compared with their transverse energy. In such a case plasma is located near the surface formed by the points of the minimum field strength on the given field line (the so-called "disk"-like plasma), and the contribution of the particles with fast drift can be expressed as

$$W = \frac{e^2}{4}\int d\mu d\Phi\ \frac{F}{\mu}\ \frac{\partial}{\partial\Phi}\left(\frac{\tilde{\varphi}^2}{B_{0\Phi}}\right), \quad (24)$$

or after the integration by parts

$$W = -\frac{e^2}{4}\int d\mu d\Phi\ \frac{\tilde{\varphi}^2}{\mu B_{0\Phi}}\ \frac{\partial F}{\partial\Phi}, \quad (25)$$

where B_0 is the minimum value of the field strength on the magnetic surface, corresponding to the flux Φ, and $B_{0\Phi} \equiv \partial B_0/\partial\Phi$.

III Generalization of the energy principle for the case: $\Gamma/m \ll \Omega_d \leq \Gamma$

The crucial point of the derivation of the expression for the energy variation in Section II (as well as in [7]) is the exploitation of the fact that the flux adiabatic invariant conserves. As is well known (see, e.g. [11]), when the frequency of the drift motion Ω_d around the axis is high as compared with the inverse characteristic time of variation of the electric and magnetic fields, then the magnetic flux $\tilde{\Phi}$ encompassed by the drift surface is an adiabatic invariant

(notation $\tilde{\Phi}$ shouldn't be confused with a flux coordinate Φ , an independent variable). In the case considered in the present paper, the magnetic field is constant; the varying is electrostatic potential φ of the perturbations. The slowness of variation of φ in the above mentioned sense (see (4)) guarantees the conservation of $\tilde{\Phi}$.

When considering the case of perturbations with high azimuthal mode number $m \gg 1$, one may encounter the situation when the drift frequency Ω_d is lower then Γ but higher than Γ/m :

$$\Gamma/m \ll \Omega_d \leq \Gamma. \tag{26}$$

The inequality $\Omega_d \leq \Gamma$ means that one cannot thoughtlessly use the traditional adiabatic invariant $\tilde{\Phi}$. However, we shall show that, under the condition $\Gamma/m \ll \Omega_d$, there exists another adiabatic invariant, similar to $\tilde{\Phi}$.

We consider the potential perturbation (not necessarily small) which is of the form (6) and changes in time with the characteristic frequency Γ. Then the inequality $\Gamma/m \ll \Omega_d$ means that the guiding centre traverses one spatial period of the system in a time that is short as compared to the time of potential variation. Just this fact is a basis for existence of the generalized adiabatic invariant.

The guiding centre motion, averaged over the fast bouncing along the field line, is governed by the following equations [12]:

$$\begin{aligned} \dot{\psi} &= -\frac{2\pi c}{e t_{\|}} J_{\Phi}, \\ \dot{\Phi} &= \frac{2\pi c}{e t_{\|}} J_{\psi}, \end{aligned} \tag{27}$$

where $t_{\|} = J_{\varepsilon}$ is the transit time between the turning points. In the context of the problem under consideration, J is a constant of motion (as well as μ).

Resolving the equation $J = const$ with respect to ε , we can find the function $\varepsilon(\psi, \Phi)$ for a given particle. By differentiating the function $J(\varepsilon(\psi, \Phi), \Phi, \psi)$ over ψ and Φ , we find:

$$\frac{\partial \varepsilon}{\partial \psi} = -\frac{J_{\psi}}{J_{\varepsilon}}, \qquad \frac{\partial \varepsilon}{\partial \Phi} = -\frac{J_{\Phi}}{J_{\varepsilon}},$$

and the equations of motion (27) acquire the Hamiltonian form:

$$\begin{aligned} \dot{\psi} &= \frac{2\pi c}{e} \frac{\partial \varepsilon}{\partial \Phi}, \\ \dot{\Phi} &= -\frac{2\pi c}{e} \frac{\partial \varepsilon}{\partial \psi}, \end{aligned} \tag{28}$$

with ψ and Φ being canonically conjugate variables and ε playing a role of Hamiltonian. As the explicit dependence of ε on t is slow in the sense that the guiding centre traverses one spatial period of the system (in coordinate ψ) in a time short as compared to the time of potential variation, equations (28) possess an adiabatic invariant

$$\tilde{\Phi}^{\star} = \int_{T_{\psi}} \Phi \; d\psi, \tag{29}$$

where integration is carried out over one spatial period T_{ψ}, equal to $2\pi/m$. Of course, for slow enough potential variations, $\Gamma \ll \Omega_d$, the integration in (29) can be extended to a full rotation of a particle around the axis, and (29) reduces to the standard flux invariant.

Now it becomes clear that the expression for the energy variation, that takes into account conditions (26), coincides with formula (22). Indeed, all calculations in this case would exactly repeat calculations, performed in Section II, with the only exclusion: $\tilde{\Phi}$ conservation requirement should be replaced by the requirement of the conservation of $\tilde{\Phi}^{\star}$. But this changing doesn't, obviously, affect formula (22), since the averaging of the expressions,

composed of function (6), either over $2\pi/m$-interval (as in (29)) or over 2π-interval (as in the expression for $\tilde{\Phi}$) would finally lead to the same result.

IV Qualitative estimates

Formulas (22), (23) define the contribution of the fast drifting particles (4) to the total energy variation; there exist also contributions, originating from plasma particles with small drift frequencies (1), whose response can be obtained in the framework of MHD approach. The corresponding additional terms represent the kinetic ($W^{(k)}$) and the potential ($W^{(p)}$) energies of the slow particles. If we restrict ourselves by the most dangerous "global" mode, the kinetic energy can be evaluated as

$$W^{(k)} \cong M_i n_c a^2 L_c \dot{\xi}^2, \tag{30}$$

where n_c, T_c, a are density, temperature and radius of plasma in the central cell; ξ is a displacement of the plasma column as a whole in the central cell. The potential energy arises from the curvature effects in the transition region and can be written in the form:

$$W^{(p)} \cong -\frac{n_c T_c a^2}{L_{tr}} \xi^2 . \tag{31}$$

The sign of $W^{(p)}$ is negative due to the unfavorable field line curvature in the transition region. As it follows from (30), (31), in the central cell itself (with the anchors being switched off) the instability developes with the characteristic growth rate given by (2).

Taking into consideration the contribution (22) of the fast particles, notice that the expression for W scales as φ^2. Since the displacement ξ of the flux tube, filled with a cold plasma, is determined by formula (5) with $\eta = -(1/c)\int \varphi dt$, we see that, if (23) is expressed in terms of ξ, it scales as

$$W \cong \frac{n_s L_s^3}{T_s} \frac{e^2 a^2 B_c^2}{c^2} \dot{\xi}^2 . \tag{32}$$

Here we have supposed the radial dimension of plasma in the stabilizer to be of the order of L_s, and have taken into account the conservation of the magnetic flux through the plasma cross-section,

$$B_c a_c^2 \sim B_s L_s^2. \tag{33}$$

As it follows from (32), W scales as a kinetic energy of the perturbations ($\sim \dot{\xi}^2$), giving contribution to $W^{(k)}$, not to $W^{(p)}$. Therefore the presence of the fast drifting particles manifests itself in changing of the "inertia" of the perturbations, not of their "rigidity".

Fast drifting particles may affect the stability in two different ways, depending on the sign of the energy W. Positive value of W gives rise to the increasing of the effective kinetic energy of the perturbations (or of their effective "inertia") that , in turn, leads to the decreasing of the growth rate Γ, but the instability still remains.

When, on the contrary, W is negative and, moreover, the following requirement is satisfied:

$$W^{(k)} + W < 0 , \tag{34}$$

the "inertia" of the perturbations becomes negative. Since the frequency of such oscillations is, obviously, real, condition (34) allows one to conclude that the system becomes stable.

Estimates (30), (32), together with (34), impose one more constraint on the plasma parameters:

$$\frac{n_s}{n_c}\frac{L_s}{L_c} > \frac{\rho_{Hs}^2}{L_s^2}\frac{B_s^2}{B_c^2},$$

where ρ_{Hs} is the Larmor radius of the hot ion in the stabilizer region.

Now it's appropriate to mark one more aspect. The stabilization due to strong FLR effects takes place in the conditions [4]

$$\Gamma \ll \frac{cT_c}{eB_c a^2}. \tag{35}$$

Combining (1), (3), (35) and taking into account (33), one can find that the assumption of the dominating FLR effects in the limit (1) is valid only if

$$T_s \ll T_c.$$

Since the latter condition is rather restrictive, it stresses all the more the actuality of the stability analysis, involving particles with fast drift frequency (4).

One more example of the magnetic configuration in which the fast drifting particles may considerably affect the stability of the plasma, is a single-cell non-paraxial mirror device of length L (with a plasma occupying a volume of the order of L^3). Let plasma consist of a thermal population with temperature T and density n, and a hot population with temperature T_* and density $n_* < n$; let also the pressure of the hot component exceed that of the cold one: $n_* T_* > nT$. For the mode of a "global" displacement one can evaluate the plasma kinetic energy (per unit volume) as

$$\left(\frac{n_* T_*}{{\Omega_d}^2 L^2} + nM\right)\dot{\xi}^2, \tag{36}$$

where ξ is a (small) plasma displacement. The first term here represents a contribution of the fast particles. The potential energy is just

$$nT(\xi/L)^2, \tag{37}$$

as fast particles do not contribute to it. If the drift frequency of the fast particles Ω_d is not too high,

$$\Omega_d < \frac{1}{L}\left(\frac{n_* T_*}{nM}\right)^{1/2},$$

the inertia of the fast particles dominates. The estimate for the growth-rate Γ is then

$$\Gamma \sim \Omega_d \left(\frac{nT}{n_* T_*}\right)^{1/2}.$$

As $n_* T_* > nT$, the growth-rate is automatically less than the drift frequency, ensuring the applicability of our analysis. So, we see that, indeed, the "inertia" of the fast drifting particles can be dominant, despite their small density.

Notice that though in the present section we have restrected ourselves with the stability analysis of the "global" mode (estimates (30), (31), (36), (37) hold just for the "global" mode perturbations), the stabilization due to the "negative inertia" effect, discussed above, makes it possible to suppress, in principle, the instability of an arbitrary azimuthal mode. However, as the "inertia" of the oscillations is negative (see (34)), the dissipative instabilities are possible.

V.1 Paraxial approximation

As it was shown in Section IV, the necessary condition for the stabilization of the flute instability due to the "negative inertia" effect is the negative definition of the energy variation W of the fast particles:

$$W < 0. \tag{38}$$

In the given magnetic field the latter inequality imposes certain restrictions on the possible profiles of the distribution function F of the fast particles.

In the present Section we consider the requirements, which the distribution function F should fit to satisfy (38) in the magnetic mirror configuration.

We start with the stability analysis of the paraxial magnetic mirror. In the framework of the paraxial approach, the plasma radial dimension should be small compared to the mirror-to-mirror distance. If we use the coordinate frame with the axis coinciding with the magnetic axis, the longitudinal invariant J, up to the terms linear in Φ, can be written as [13]

$$J = J^{(0)}(\varepsilon, \mu) + \Phi J^{(1)}(\varepsilon, \mu).$$

The neglect of the higher order terms in Φ is justified in the paraxial region. Since the variation of F in Φ has a small scale-length (the plasma radius is assumed to be much smaller than the device length!), first term in (23) is dominant, and so expression for W reduces to a simplified form:

$$W = \frac{1}{2} e^2 \int d\varepsilon d\mu d\Phi \ \frac{J_\varepsilon^{(0)}}{J^{(1)}} \ \tilde{\varphi}^2 \ \frac{\partial F}{\partial \Phi}. \tag{39}$$

Since the derivative $\partial F/\partial \Phi$ defines the sign of the diamagnetic frequency, one can conclude from (10), (39), taking into account $J_\varepsilon^{(0)} > 0$, that the energy variation would be negative for those particles whose directions of the curvature-induced and diamagnetic drifts coincide, and would be positive in the opposite case.

As an example of the stability analysis of the concrete plasma configuration, we examine the stability of the hot "disk"-like plasma located in the middle plane of the magnetic mirror. The energy variation of such a plasma is given by formulas (24), (25). The paraxial expansion of the magnetic field in the middle plane can be presented in the form [13]:

$$B_0 = \mathcal{B} - \frac{1}{4\pi} \frac{\mathcal{B}''}{\mathcal{B}} \Phi, \tag{40}$$

where $\mathcal{B}$ is the field strength on the axis and prime denotes the derivative along the axis. Note that $\mathcal{B}'' > 0$, since the magnetic field has a minimum in the middle plane. Inserting (40) into (25), one obtains:

$$W = \pi e^2 \frac{\mathcal{B}}{\mathcal{B}''} \int d\mu d\Phi \ \frac{\tilde{\varphi}^2}{\mu} \ \frac{\partial F}{\partial \Phi}. \tag{41}$$

As it follows from the latter expression, condition (38), necessary for the stabilization, is satisfied for

$$\frac{\partial F}{\partial \Phi} < 0.$$

V.2 Stability of the "global" mode

Expression (41) can be further simplified in the case of a global mode when the potential φ is given by

$$\varphi = \varphi_0 \sqrt{\Phi} \cos \psi. \tag{42}$$

After the substitution of (40), (42) into (24), one can find

$$W = -\pi e^2 \varphi_0^2 \frac{\mathcal{B}}{\mathcal{B}''} \int d\mu d\Phi \ \frac{F}{\mu}. \tag{43}$$

For $\mathcal{B}'' > 0$, inequality (38) is satisfied, and the "negative inertia" stabilization is realizable. We see that the perturbation of the fast particle kinetic energy is negative irrespectively to possible presence of regions (in the radial direction) where $\partial F/\partial \Phi$ is positive.

Up till now in the present Section the discussion has been restricted by the framework of the paraxial plasma configurations. To illustrate the influence of non-paraxiality on the stability of the plasma with fast particles, we turn to the stability analysis of the hot "disk"-like plasma, localized in the middle

plane of the magnetic mirror produced by two point-like co-axial coils Fig.1 (cf. [13]).

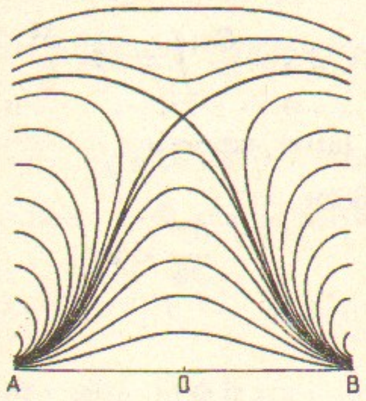

Fig. 1. Field lines of the magnetic mirror produced by two co-axial point-like coils. Bold line marks the separatrix, that passes through the point of the zero field strength.

The magnetic field in the midplane of such a mirror can be written in the form

$$B_0 = B_* \frac{(2-x)}{2(x+1)^{5/2}}, \tag{44}$$

where $x = 4r^2/L^2$, r is a radius in the middle plane, L is a distance between dipoles and B_* is the field strength in the middle point between them. The magnetic flux can be expressed in terms of x as follows:

$$\Phi = \Phi_s \frac{3^{3/2}}{2} \frac{x}{(1+x)^{3/2}}, \tag{45}$$

where Φ_s denotes the flux, corresponding to the separatrix that bounds the region of the adiabatic confinement (see Fig.1).

Now we examine the sign of the energy variation W for the "global" mode perturbation (42). Consider the plasma ring, involving particles located on

the distance R_0 from the axis in the equilibrium state. Inserting (42), (44), (45) into (24), after elementary analysis one can obtain, that the energy variation W occurs to be negative for the plasma rings with radii

$$\frac{R_0}{R_s} < 0.72, \tag{46}$$

where R_s is the separatrix radius, while for radii R_0, being outside the interval (46), the energy W has a positive sign. Hence, non-paraxial effects lead to the violation of the condition (38), **and** so the region of distant radii gives unfavorable contribution to W in the sense of the possibility of the "negative inertia" stabilization.

VI Application to the beam-plasma neutron source

Our results can be of some interest for the development of the beam-plasma neutron source (BPNS) [10]. In this system a relatively short mirror cell is filled with a cold tritium plasma which serves as a target for the deuterons with the mean energy of ~60keV, which are produced by NB injection and confined in the same mirror cell. In order to reduce the heat losses through the cold plasma electrons, there is envisaged the use of the long solenoidal sections with gradually decreasing magnetic field between the mirror cell and the end-walls: as the target plasma is a collisional one and the heat flux is determined by the thermal conductivity, this arrangement indeed reduces the heat flux. Another implication of the using of these long sections is a considerable increase of the inertia of the flute perturbations. As one can

easily show [15], for the global mode the kinetic energy can be evaluated as

$$\dot{\xi}^2 \pi a_0^2 B_0^2 \int \frac{Mndl}{B^2},$$

where ξ is a plasma displacement in the equatorial plane of the device, B_0 and a_0 are magnetic field strength and plasma radius in this plane, respectively. In the BPNS conditions the ionized and the neutral components in the flaring of the magnetic field are strongly coupled via charge exchange and ellastic collisions, so that n here is the total density of the charged and neutral components.

Denoting the half-length of the mirror cell as L, one can obtain the following expression for the potential energy:

$$\xi^2 (n_* T_* / L^2) \pi a_0^2 L,$$

where the subscript "$*$" refers to the high energy particles.

The growth-rate Γ of the flute perturbations is equal to

$$\frac{1}{L}\left(\frac{T_*}{M}\right)^{1/2}\left(\frac{n_* L}{B_0^2 \int \frac{ndl}{B^2}}\right)^{1/2},$$

while the drift frequency of the fast ions is

$$\Omega_d \sim \frac{\rho_*}{L^2}\left(\frac{T_*}{M}\right)^{1/2}.$$

Here ρ_* denotes the Larmor radius of the hot particle. After the substitution of the numerical values of all variables (see [10]), it turns out that

$$\frac{\Gamma}{\Omega_d} \sim 0.1,$$

that is, the contribution of the fast ions to the potential energy of the flute perturbations in fact is zero. This should allow the change of the magnetic

configuration from Yin-Yang to axisymmetric one, with the corresponding simplification of design of the neutron source.

VII Conclusions

In the present paper we have analyzed the stability of the axisymmetric mirror device with respect to purely electrostatic flute perturbations. We have investigated the scheme consisting of the long central cell conjugated with the end stabilizer that contains hot plasma. One of the main results of the paper is the establishing of the fact, that a population of hot particles with fast azimuthal drift (see (4)) may give a favorable contribution to the stability of the system as the whole. The reason is that fast drifting particles affect not a potential energy of MHD perturbations but rather their kinetic energy, and in the case, when the effective kinetic energy is found to be negative, the stabilization takes place.

Another system, to which our results can be applied, is a two-component plasma consisting of a cold dense background and a minority of hot particles that determines the plasma pressure (like in some versions of mirror-based neutron sources [10, 16]). At high enough density of a cold component, condition (4) can easily be met.

We have also showed that the energy principle, formulated in [9], can be generalized to the intermediate frequency interval $\Gamma/m \ll \Omega_d \leq \Gamma$ for the azimuthal modes with high numbers m. Therefore all the conclusions, concerning the stabilization due to the "negative inertia" effect, can be transformed to these conditions as well.

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A Energy principle for the "disk"-like plasma

Consider the limit of the zero longitudinal pressure of the hot plasma, $p_\parallel = 0$, when the transverse energy of hot particles greatly exceeds their longitudinal energy. In the equilibrium state these particles perform small bounce oscillations round minimum field point, so that

$$\varepsilon - \mu B_0 \ll \mu B_0, \tag{47}$$

B_0 marks the minimum value of the field strength on the given field line. Using expansion of B along the field line in the vicinity of B_0,

$$B \simeq B_0 + \tilde{B} l^2,$$

one can carry out the integration in (8) explicitly:

$$J = \pi\sqrt{2M}\ \frac{\Delta\varepsilon}{\sqrt{\mu\tilde{B}}}, \tag{48}$$

with $\Delta\varepsilon = \varepsilon - \mu B_0$. Now it would not be difficult to calculate with sufficient accuracy ε- and Φ- derivatives of J:

$$J_\varepsilon = \frac{J}{\Delta\varepsilon}, \qquad J_\Phi = -\mu B_{0\Phi}\frac{J}{\Delta\varepsilon},$$

$$J_{\varepsilon\varepsilon} = 0, \qquad J_{\varepsilon\Phi} = -\frac{\tilde{B}_\Phi}{\tilde{B}}\frac{J}{\Delta\varepsilon}, \tag{49}$$

$$J_{\Phi\Phi} = -\mu B_{0\Phi\Phi}\frac{J}{\Delta\varepsilon} + \mu B_{0\Phi}\frac{\tilde{B}_\Phi}{\tilde{B}}\frac{J}{\Delta\varepsilon}.$$

Here we have taken into account, that $\Delta\varepsilon/\mu B_0$ is a small parameter according to (47). Substitution of (49) into (22) leads to the following expression for the energy variation:

$$W = \frac{e^2}{4}\int d\mu d\Phi\ \frac{F}{\mu}\ \frac{d}{d\Phi}\left(\frac{\tilde{\varphi}^2}{B_{0\Phi}}\right). \tag{50}$$

Note that there is no integration over $d\varepsilon$ in (50). The reason is that in the case under consideration, the distribution function F depends on ε approximately in a δ-functional way, $F \sim \delta(\varepsilon - \mu B_0)$, and so the integration over $d\varepsilon$ can be done explicitly.

References

[1] *D.D. Ryutov* // Plasma Physics and Controlled Fusion, **28**, 191 (1986).

[2] *V.V. Arsenin* // Itogi Nauki i Tekhniki, Fiz. Plazmi, vol. 8, p.49 (1988) (in Russian).

[3] *D.D. Ryutov* // Sov. Phys.-Uspekhi, **31**, 300 (1988).

[4] *M.N. Rosenbluth, N.A.Krall, N.Rostoker* // Nucl. Fusion, Suppl. Pt. 1, 143 (1962).

[5] *M.D. Kruskal, C.R. Oberman* // Phys. Fluids, **1**, 275 (1958).

[6] *V.P. Nagornyj, D.D. Ryutov, G.V. Stupakov* // Preprint N-83-74, Institute of Nuclear Physics, Novosibirsk (1983).

[7] *J.W. Van Dam, M.N. Rosenbluth, Y.C. Lee* // Phys. Fluids, **25**, 1349 (1982).

[8] *I.M. Lansky, D.D. Ryutov* // Preprint N-92-91, Budker Institute of Nuclear Physics, Novosibirsk (1992).

[9] *I.M. Lansky, D.D. Ryutov* // JETP Letters, **57**, No.1, 30 (1992).

[10] *F.H. Coensgen, T.A. Casper, D.L. Correl, et al.* // Rep. UCRL-97280, Rev. 1, Lawrence Livermore National Laboratory (1987).

[11] *T.G. Northrop, E. Teller* // Phys. Rev., **117**, 215 (1960).

[12] *A.I. Morozov, L.S. Solov'ev* // Reviews of Plasma Physics, **2**, Consultants Bureau, New York (1966).

[13] *D.D. Ryutov, G.V. Stupakov* // Reviews of Plasma Physics, **13**, Consultants Bureau, New York — London, 93 (1987).

[14] *D.D. Ryutov, G.V. Stupakov* // Sov.J. Plasma Phys., **12**, 815 (1986).

[15] *V.P. Nagornyj, D.D. Ryutov, G.V. Stupakov* // Nucl. Fusion, **24**, No. 11, 1421 (1984).

[16] *V.V. Mirnov, V.P. Nagornyj, D.D. Ryutov* // Preprint N-84-40, Institute of Nuclear Physics, Novosibirsk (1984).

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Generalized Energy Principle for Flute Perturbations in Axisymmetric Mirror Machines

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Обобщенный энергетический принцип для желобковых возмущений в осесимметричных открытых ловушках

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