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LIMITATIONS ON CURRENT RIPPLE OF THE POWER SUPPLIES FOR THE SSC BENDING MAGNETS

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Limitations on Current Ripple of the Power Supplies for the SSC Bending Magnets

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Abstract

Noise and ripple in the bending magnets of large proton collider cause the beam emittance growth and the luminosity degradation. The emittance growth due to voltage ripple of the bending magnets power supplies is studied. The role of the collider transverse feedback system is shown to be very important to facilitate the requirements to value of ripple. The longitudinal emittance growth due to slow variations of power supply current are studied as well.

Contents

1. Introduction

Noise and ripple in the magnetic field of a storage ring produce the beam betatron motion which leads to the emittance growth due to the betatron tune spread in the beam and, consequently, leads to the luminosity reduction. It is especially dangerous for the Superconducting Super Collider because of very low revolution frequency and beam emittance. The main sources of this external perturbation are the transverse displacements (oscillations) of quads and ripple and noise in power supplies of magnets and correctors. This article is devoted to the analyses of power supply ripples and noises on beam dynamics and stands limits on the acceptable amplitude of power supply ripples and noises.

The Collider will have a lot of different power supplies for the magnets. They are 10 powerful sources for the regular dipoles and quadrupoles, some tens of power supplies for quads of the utility sections and the interaction regions, about 5000 small power supplies for correctors. Potentially, ripples of all these power supplies will affect the beam and will lead to the emittance growth. Because of a huge number it will be very difficult to identify the real sources of the emittance growth. Therefore all measures have to be undertaken to decrease the harmful influence of power supply ripples and noises. To do this we consider the next measures should be undertaken.

1. The powerful transverse feedback system should be used to suppress the emittance growth which originates from dipole beam kicks. This can reduce the emittance growth rate in 100-500 times.
2. The collider tunes should be chosen between power line harmonics that practically removes any their influence on beam dynamics because of small betatron tune spread in the beam.
3. To suppress the current ripple from the power supplies, the system of passive filters should be used.
4. The use of the damping resistors in parallel to the superconducting magnets dumps the ripple wave propagation in the string of magnets that decreases the beam kicks from the magnets.

2. Transverse Emittance Growth and its Suppression by the Feedback System

There is a lot of different processes that can lead to the emittance growth. First, we consider the emittance growth due to random dipole kicks when all particles are affected by the same sequence of transverse kicks. This excites the betatron coherent motion which, due to unavoidable betatron tune spread, will pump up the beam emittance. The sources of these dipole kicks are field fluctuations and ripples in dipoles and transverse oscillations in positions of quadrupoles.

2.1. The Emittance Growth due to Noise and Ripple.

The emittance growth due to external dipole noise and suppression of this emittance growth by the transverse feedback system was studied in the refs.[1] and [2]. It was shown that in the general case where one can neglect coupling between the vertical and horizontal motion, and the spectral density of external perturbation is a smooth function of frequency (i.e. the spectral density does not change significantly in a frequency band of the betatron tune spread), the emittance growth rate is equal to:

$$\left(\frac{d\epsilon}{dt}\right)_0 = \frac{\omega_0^2}{4\pi}\sum_{n=-\infty}^{\infty}\beta S(\omega_n), \qquad \omega_n = \omega_0(\nu - n). \tag{2.1}$$

Here ω_0 is the revolution frequency, ν is the betatron tune, $S(\omega)$ is the spectral density of beam kick summed over one turn and referenced to β-function β. This spectral density is normalized so that the correlation function of the beam kick is

$$K(\tau_1-\tau_2) \equiv \langle\theta(\tau_1)\theta(\tau_2)\rangle = \int_{-\infty}^{\infty} S(\omega)e^{i\omega(\tau_1-\tau_2)}d\omega \ , \tag{2.2}$$

where $\theta(\tau)$ is the angle kick summed over one turn ($\theta = dx/ds$ for horizontal and $\theta = dy/ds$ for vertical directions) excited by magnetic field fluctuations in particle motion. One can express this "summed" spectral density through the cross spectral density of angle kicks from different magnets as

$$\beta S(\omega) = \sum_{i,j=1}^{N}\sqrt{\beta_i\beta_j}S_{ij}(\omega)\cos(\mu_i - \mu_j - \omega\tau_{ij}) \ . \tag{2.3}$$

Here N is a number of magnets, β_i and β_j are the β-functions in magnets i and j, μ_i-μ_j is the betatron phase advance between the magnets i and j, τ_{ij} is the time of flight from the magnet i to the magnet j, S_{ij} is the cross spectral density of angle kicks from the magnets i and j. This cross spectral density is bound up with the cross correlation function of angle kicks by the expression

$$K_{ij}(\tau_1-\tau_2)=\langle\theta_i(\tau_1)\theta_j(\tau_2)\rangle=\int_{-\infty}^{\infty}S_{ij}(\omega)e^{i\omega(\tau_1-\tau_2)}d\omega\ , \qquad (2.4)$$

where $\theta_i(\tau)$ and $\theta_j(\tau)$ are angles excited by kicks of the magnets i and j. Eq.(2.1) is justified for both the vertical and horizontal planes. It is important to note that the coupling between the vertical and horizontal motions will redistribute the emittance growth between both planes. In the SSC case, the beam has equal vertical and horizontal emittances, so the emittance growth rates should be equal for both transverse planes

$$d\varepsilon/dt=((d\varepsilon/dt)_x+(d\varepsilon/dt)_y)/2.$$

In Eq.(2.1) one can see that only resonance frequencies contribute to the emittance growth. As a rule, the main contribution is determined by the lowest frequency $\omega_\beta=\omega_0 min([\nu],1-[\nu])$ because the spectral density of perturbation decreases very quickly with frequency. Here $[\nu]$ is the fractional part of the betatron tune.

2.2. The Emittance Growth due to Harmonic Perturbation.

In the case where the harmonic perturbation (ripple) is added to the general noise, the sharp (δ-function) peaks appear in the spectral density. If there is no overlapping between a harmonic of the perturbation frequency and betatron frequencies of particles this ripple (or harmonic perturbation) does not produce any emittance growth as follows from Eq.(2.1). But in the case where perturbation frequency is inside the betatron tune spread the general picture is much more complicated than it was considered in Section (2.1).

Let only harmonic perturbation affect the beam. Then resonance particles will increase their amplitudes and should go out of resonance because of tune dependance on amplitude. This means that after short time the emittance increase should be stopped by nonlinearity. But in real conditions there are some effects which will prevent from it. They are: the general noise which will push new particles to the resonance; slow fluctuations in particle tunes due to final accuracy of beam aiming and slow current fluctuations in magnets; a change of counter-rotating beam profile (due to its emittance growth) that also changes the tunes of

particles; and some other effects, such as intrabeam scattering, etc.

Let us consider a simple estimate which determines the upper limit of the emittance growth rate. We will neglect the coupling between the vertical and horizontal degrees of freedom and the motion nonlinearity. To simplify solution we introduce the complex variable

$$Z=\frac{x}{\sqrt{\beta}}+i\beta\frac{d}{ds}\left(\frac{x}{\sqrt{\beta}}\right), \qquad (2.5)$$

which changes as

$$Z_n=Z_0e^{-2\pi i\nu n} \qquad (2.6)$$

for the unperturbed betatron motion. Here x stands for the particle deviation with respect to the closed orbit, β is the β-function and s is the path length along the orbit, ν is the betatron tune and n is the turn number. Let the beam be influenced by harmonic perturbation so that on turn n it gets an angle kick equal to $\theta_e\cos(2\pi\nu_e n)$. Thus, one gets an increment for the variable Z equal to

$$\Delta Z_n=i\theta_e\sqrt{\beta_e}\frac{e^{2\pi i\nu_e n}+e^{-2\pi i\nu_e n}}{2}\ , \qquad (2.7)$$

where θ_e and ν_e are the amplitude and frequency of the perturbation, β_e is the β-function at the point of perturbation. Using Eqs.(2.6) and (2.7) and taking into account the motion linearity one gets that after n turns the particle coordinate is equal to

$$Z_n=Z_0e^{-2\pi i\nu n}+\sum_{k=0}^{n}\Delta Z_ke^{-2\pi i\nu(n-k)}=Z_0e^{-2\pi i\nu n}+$$
$$+\sqrt{\beta_e}\frac{\theta_e}{2}i\sum_{k=0}^{n}\left(e^{2\pi i\nu_e k}+e^{-2\pi i\nu_e k}\right)e^{-2\pi i\nu(n-k)}=\left(Z_0+\Sigma_n(\nu)\right)e^{-2\pi i\nu}\ , \qquad (2.8)$$

where $Z_0=|Z_0|e^{i\psi}$ is the particle initial coordinate and

$$\Sigma_n(\nu)=\sqrt{\beta_e}\frac{\theta_e}{2i}\left(\frac{1-e^{-2\pi i(\nu+\nu_e)n}}{1-e^{-2\pi i(\nu+\nu_e)}}+\frac{1-e^{-2\pi i(\nu-\nu_e)n}}{1-e^{-2\pi i(\nu-\nu_e)}}\right). \qquad (2.9)$$

After averaging in the initial phase ψ

$$\langle Z_n^2\rangle_\psi=|Z_0^2|+|\Sigma_n^2(\nu)|. \qquad (2.10)$$

and averaging over the function distribution, one has the change in the r.m.s. beam emittance after n turns

$$\delta\epsilon(n)=\frac{1}{2}(\langle Z_n^2\rangle-\langle Z_0^2\rangle)=\frac{1}{2}\int|\Sigma_n^2(\nu)|f(\nu)\,d\nu\ , \tag{2.11}$$

where $f(\nu)$ is the distribution function over the betatron tune. For a very large number of turns, the second resonance addend in Eq.(2.9) will produce the main contribution in the integral (2.11). If $n \gg 1/\Delta\nu$, then $|\Sigma_n^2(\nu)|$ can be replaced by the δ-function and the integral (2.11) can be easily calculated. In this case one has

$$\delta\epsilon(n)=\frac{\beta\theta_e^2}{8}f(\nu_e)n\ , \tag{2.12}$$

where $\Delta\nu$ is the betatron tune spread. Taking into account that in the collider mode the maximum of the distribution function is about 1.75ξ (see Appendix A), we finally have the emittance growth rate in the resonance

$$\left(\frac{d\epsilon}{dt}\right)_0\approx 0.22\frac{\beta\theta_e^2}{\xi}f_0 . \tag{2.13}$$

Here ξ is so called interaction parameter

$$\xi=\frac{e^2N_{particle}}{4\pi E\epsilon}\ , \tag{2.14}$$

$N_{particle}$ is a number of particles in the bunch, E is the particle energy.

One can see that Eq.(2.13) can be derived from (2.1) if the spectral density in Eq.(2.1) will be equal to

$$S(\omega_0\nu_e)\approx 0.44\frac{\theta_e^2}{\omega_0\xi}\ . \tag{2.15}$$

It looks like that the spectral density of the harmonic perturbation, which is proportional to $\delta(\omega-\omega_0\nu_e)$, has to be smoothed within a frequency band of about the betatron tune spread.

2.3. Emittance Growth Suppression by the Feedback System.

The transverse feedback system will damp coherent betatron oscillations what will suppress the emittance growth due to dipole kicks of the beam. This process was carefully studied in refs.[1] and [2] and we will use the results of this

study here. The numerical simulations showed that the emittance growth rate can be approximated as[2]

$$\frac{d\epsilon}{dt}\approx\xi^2\left(\frac{3.3}{g^2+3.3\xi^2}+67\right)\left(\frac{d\epsilon}{dt}\right)_0\ ,\qquad \xi<0.1,\quad g\le 0.5\ , \tag{2.16}$$

where $(d\varepsilon/dt)_0$ is the emittance growth rate without feedback system, $g=2\lambda/f_0$ is the dimensionless decrement of the feedback system.

To demonstrate an accuracy of the given above analytical estimate a comparison of the numerical simulation with prediction of the estimate is shown in Figure 1. There are two curves. Curve 1 is built for the case when the harmonic perturbation only influences the beam. One can see that if there is no overlapping between frequencies of particles and the external noise frequency the emittance growth rate is close to zero. In reality it is even negative because of stochastic cooling in the numerical model[2]. If we subtract the stochastic cooling effect the emittance growth rate will be equal to zero within accuracy of simulations as predicted by the analytical theory. For curve 2 the "white" noise was added to the harmonic perturbation. Because of small spectral density its influence is negligible for resonant tunes but outside of the resonance this addend determines the emittance growth rate which value does not depend on the tune. To build curve 3 equation (2.16) was used where the emittance growth rate $(d\varepsilon/dt)_0$ was fitted by the below equation

$$\left(\frac{d\epsilon}{dt}\right)_0=\frac{1}{2}Af_0\frac{\beta\theta_e^2}{8}f(\nu_e)\ ,\quad f(\nu_e)=\frac{\exp\left(-\frac{(\nu-\nu_e-0.46\xi)^2}{2(\kappa\xi)^2}\right)}{\sqrt{2\pi}(\kappa\xi)}\ , \tag{2.17}$$

following from Eq.(2.12). The coefficient 1/2 appears here because the excitation in simulation was applied in horizontal direction only. The constants A and κ are the fitting parameters (A=0.6, κ=0.15). As one can see the simulation gave 10% smaller results than the considered above estimate. The coincidence is much better than one should expect.

For the collider the feedback system with ultimate parameters g~1 can suppress the emittance growth rate by about three orders of magnitude.

2.4. Emittance Growth due to Ripple in Quadrupoles.

The random change of quadrupole focusing strength produces transverse kicks of particles displaced from closed orbit. It also causes the transverse emittance growth. This process was studied in ref.[3]. As follows from results of

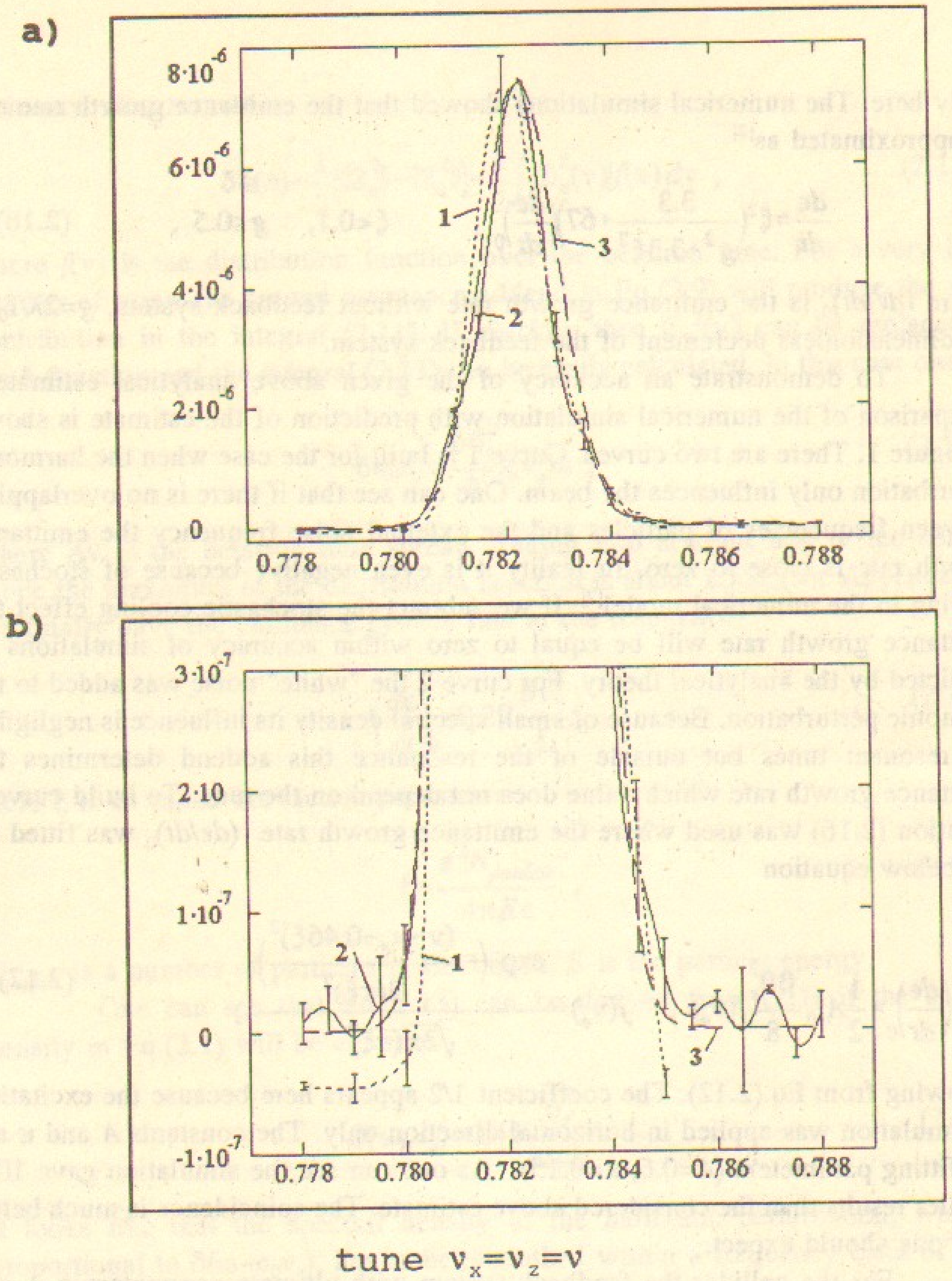

Figure 1. Dependence of the emittance growth rate per turn $d\varepsilon/dn$ on the collider tune. Simulation were done for the number of particles equal to 8190, number of turns equal to 8190, ξ=0.005, ν_e=0.78, g=0.2, θ_e=0.01. *Curve 1* - without the white noise; *curve 2* - with the white noise of r.m.s. amplitude Δ=0.05; *curve 3* - a fitting of the curve described by Eq.(2.17) to curve 2: A=0.6 and κ=0.15. The plots *a)* and *b)* show the same curves plotted in different scales.

this work in the case of noise in one quadrupole and for the gaussian distribution of particles in the phase space the emittance growth rate is equal to

$$\frac{1}{\epsilon}\frac{d\epsilon}{dt}=\frac{\omega_0^2}{4\pi}\sum_{n=-\infty}^{\infty}\left(\frac{\beta}{F}\right)^2 S_q(\omega_{q_n}), \qquad \omega_{q_n}=\omega_0(2\nu-n). \qquad (2.18)$$

Here F and β are the focusing strength of the quadrupole and β-function in its location, $S_q(\omega)$ is the spectral density of the noise normalized so that relative r.m.s. fluctuations of the focusing strength are equal to

$$\left\langle\frac{\Delta F^2}{F^2}\right\rangle=\int_{-\infty}^{\infty}S_q(\omega)d\omega. \qquad (2.19)$$

Similarly to the case of dipole excitation the resonance frequencies only produce an emittance growth. But unlike in the dipole case the resonance frequencies are on side bands of the double betatron frequency.

If there is a lot of quadrupoles in the ring with independent noises in them one should replace in equation (2.18)

$$\frac{\beta^2}{F^2}S_q(\omega)\to\sum_{j=0}^{N_q}\frac{\beta_j^2}{F_j^2}S_{q_j}(\omega), \qquad (2.20)$$

where N_q is a number of quadrupoles, β_j and F_j are the β-function in the location of the quadrupole with number j and its focusing strength.

For harmonic perturbation we can do the same estimate as for the dipole case (see Section 2.2). Let the frequency of the external perturbation $\nu_e\omega_0$ have an overlapping with a sideband of particle betatron frequencies $[(2\nu-n)\omega_0,(2\nu-n+\xi)\omega_0]$ and the focusing strength of the quadrupole is changed as

$$\frac{\Delta F}{F}=\Delta_F\cos(\nu_e\omega_0 t). \qquad (2.21)$$

Averaging the spectral density of the perturbation over betatron tune spread by the same way as in Eq.(2.15) we have an effective spectral density equal to

$$S_F\approx\frac{\Delta_F^2}{2\xi\omega_0}. \qquad (2.22)$$

After substituting Eq.(2.22) into Eq. (2.19) one has an upper limit of the emittance growth rate for the case of the resonance harmonic perturbation in the quadrupole field

$$\frac{1}{\epsilon}\frac{d\epsilon}{dt} \approx \frac{\omega_0}{8\pi\xi}\left(\frac{\beta}{F}\right)^2 \Delta_F^2 \ . \qquad (2.23)$$

Note, that this is the upper limit in the sense described in Section 2.2.

It is important that the emittance growth due to the quadrupole noise cannot be suppressed by the general transverse feedback system which damps the dipole motion only.

2.5. Limitations of Noises and Ripples for the Collider.

The most rigorous requirements on the external perturbations are for the collider mode operation when a large beam life time is required. We put here the limit on the emittance growth time $\varepsilon/(d\varepsilon/dt)$ equal to 24 hours, that is ~2 times larger than the damping time due to synchrotron radiation (SR) and means that beam heating is small in comparison with the SR cooling. The limitations of the external noise ripple are shown in Table 2 and relevant collider parameters are shown in Table 1.

Table 1
Main Parameters of the Collider

Energy	E	20 GeV
Revolution frequency	f_0	3.441 kHz
R.M.S. Emittance	ε	0.047 nm
Average β-function	β	≈200 m
Damping time due to SR	$\varepsilon/(d\varepsilon/dt)$	13 hour
Head-on beam-beam tune shift for 4 IP	ξ	0.0036

We show in Table 2 the effective spectral density of angle kicks summed over one turn

$$S_{eff} = \sum_{n=-\infty}^{\infty} S(\omega_n) \qquad (2.24)$$

for the case of noise (see (2.1)-(2.4)) and the summed over one turn kick θ_e (see

(2.7)) for harmonic perturbation. For convenience in the case of noise we also show the effective r.m.s. kick value for the white noise

$$\theta_{r.m.s.} = \sqrt{S_{eff}\omega_0} \ . \qquad (2.25)$$

For the beam storage and acceleration the requirements to noises and ripples are not so stringent because of 10 times larger emittance (the same normalized emittance) and almost 10 times smaller limit on the emittance growth time (the storage time is ~1 hour). It means that the relative value of noises $\Delta B/B$ can be ~10 times larger or, taking into account 10 times smaller energy, it means that the absolute value of perturbation ΔB can be of the same level.

Table 2
Limitation of Summed over one Turn Kick for the Collider

	S_{eff} [nrad²·s]	$\theta_{r.m.s.}$ for white noise or θ_e for ripple [nrad]
General noise		
without feedback system	$7\cdot10^{-8}$	0.04
with feedback system	$3\cdot10^{-5}$	0.8
Ripple		
without feedback system		0.004
with feedback system		0.09

3. Transfer Function from Power Supply Voltage Ripple to the Beam Kick.

There will be 10 power supply units distributed along the ring [4]; each of them covers a sector of average one-tenth of circumference. One sector contains 96 half cells. Each half cell consist of 5 dipoles and 1 quadrupole. All together there are 480 dipoles and 96 quadrupoles in each magnet string. The electrical circuit of a standard sector is shown in Figure 2. To decrease a bus voltage during a quench the power buses change their places each half cell.

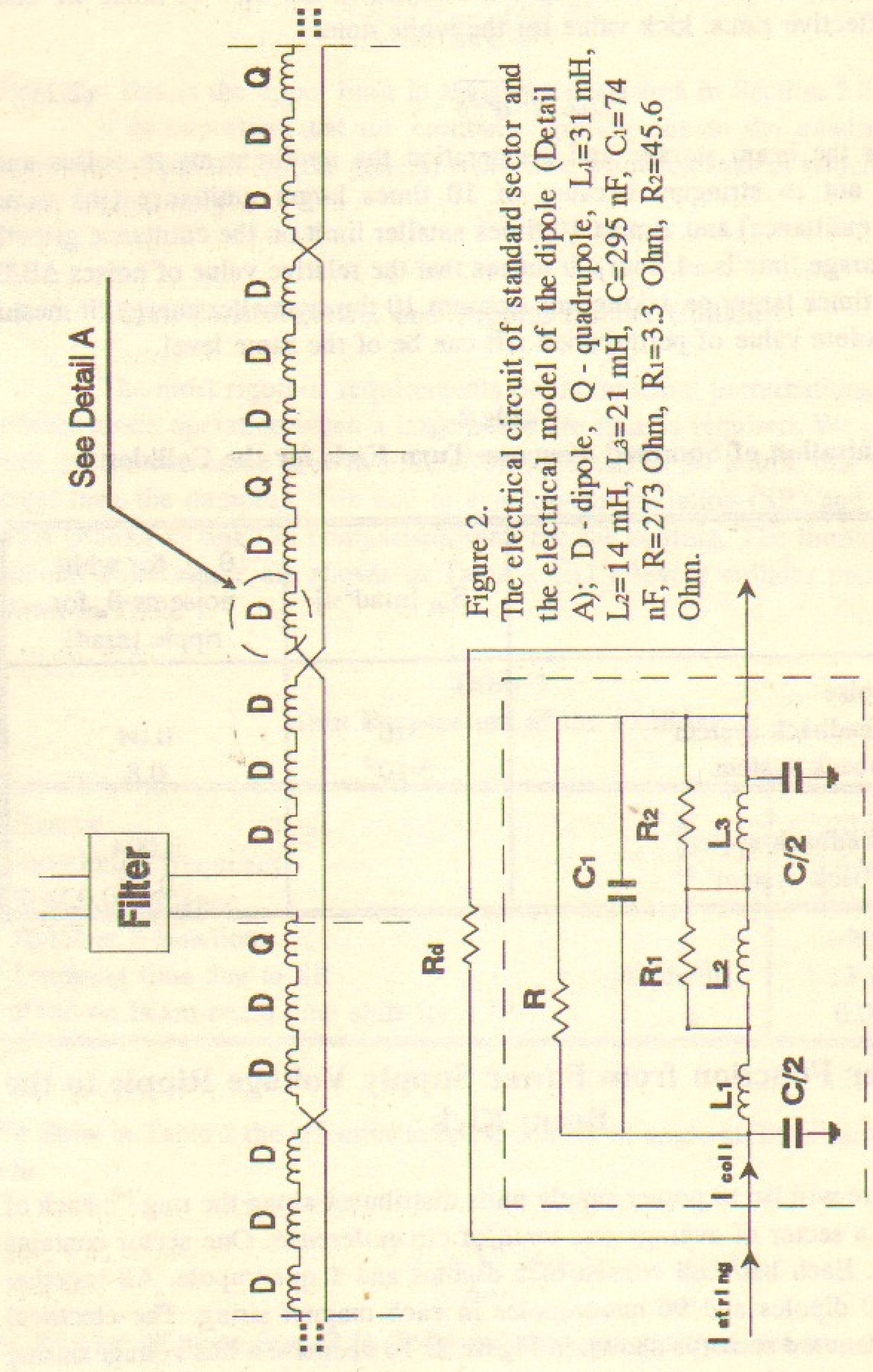

Figure 2.
The electrical circuit of standard sector and the electrical model of the dipole (Detail A); D - dipole, Q - quadrupole, L_1=31 mH, L_2=14 mH, L_3=21 mH, C=295 nF, C_1=74 nF, R=273 Ohm, R_1=3.3 Ohm, R_2=45.6 Ohm.

3.1. Propagation of Ripple Wave in a Dipole String

Because of an inductance of superconducting magnets and capacity between magnet coils and ground the magnet string looks like a transmission line. The electrical model of this transmission line was developed in ref.[5] and is shown on Figure 2. To increase the wave damping along string the damping resistors R_d will be connected across each magnet in parallel with series impedance. Below we will neglect quadrupole influence on wave propagation because its inductance is much smaller than for dipole.

For further analysis let us introduce the following string impedances: the impedance of infinite string Z_∞ and the impedances of differential Z_d=U_d/I and common Z_c=U_c/I modes for a string of finite length. Here for infinite string the voltage is supplied between the bus and the ground; for differential mode the voltage is supplied asymmetrically between buses $\pm U_d$; and for common mode the voltage is supplied between the ground and both buses. In the given definition the impedance of differential mode is two times smaller than that in the general definition. We use it for consistency with other two definitions. The dependencies of the modules of these impedances on frequency are shown in Figure 3. One can see that for high frequencies (f>30 Hz) when the damping length is smaller than the string length all impedances are equal. It means that in the frequency range of the betatron motion (f≥500 Hz) we can always use the model of infinite string for calculating the interaction between the ripple wave and the beam. So, for infinite string the complex current in the magnet with a number k is equal to

$$I_k=\frac{U}{Z_\infty(\omega)}\exp\left(\left(i\chi(\omega)-\frac{1}{N_d(\omega)}\right)k\right), \qquad k=0,\ 1,\ 2,\ \ldots \qquad (3.1)$$

where $N_d(\omega)$ is a damping length expressed in a number of dipoles and $\chi(\omega)$ is the phase advance per dipole. The dependencies of the damping length and the phase advance on frequency are shown in Figure 4. Formulas for the calculation of the string impedances are obtained in Appendix B.

3.2. The Amplification Factor

To find the beam kick summed over whole string we shall use the complex variable introduced in Eq.(2.5). Then the summed kick referenced to the drive point is

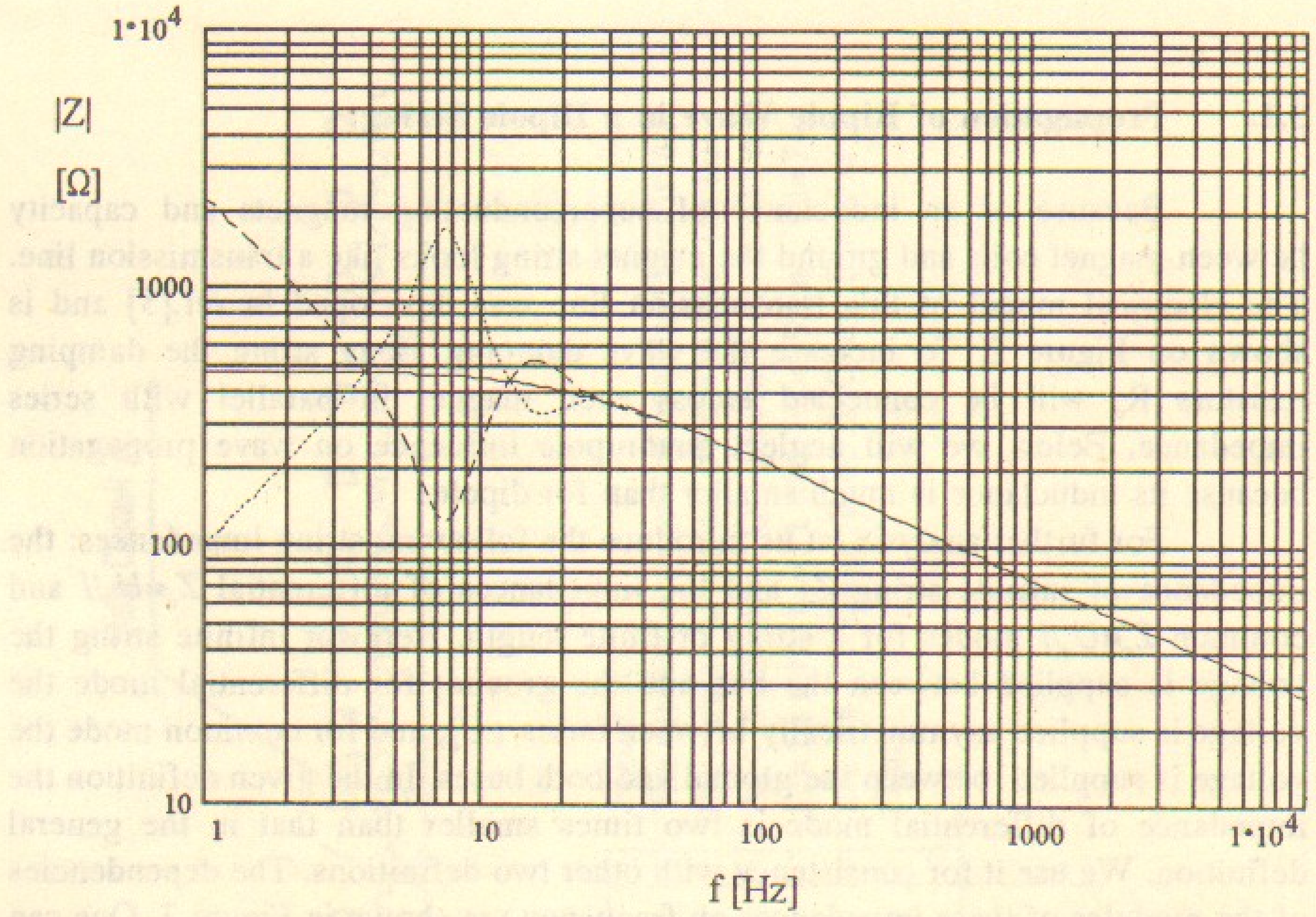

Figure 3. The dependencies of modules of the string impedances for differential mode (dotted line), common mode (dashed line) and infinite string (solid line); R_d=10 Ω.

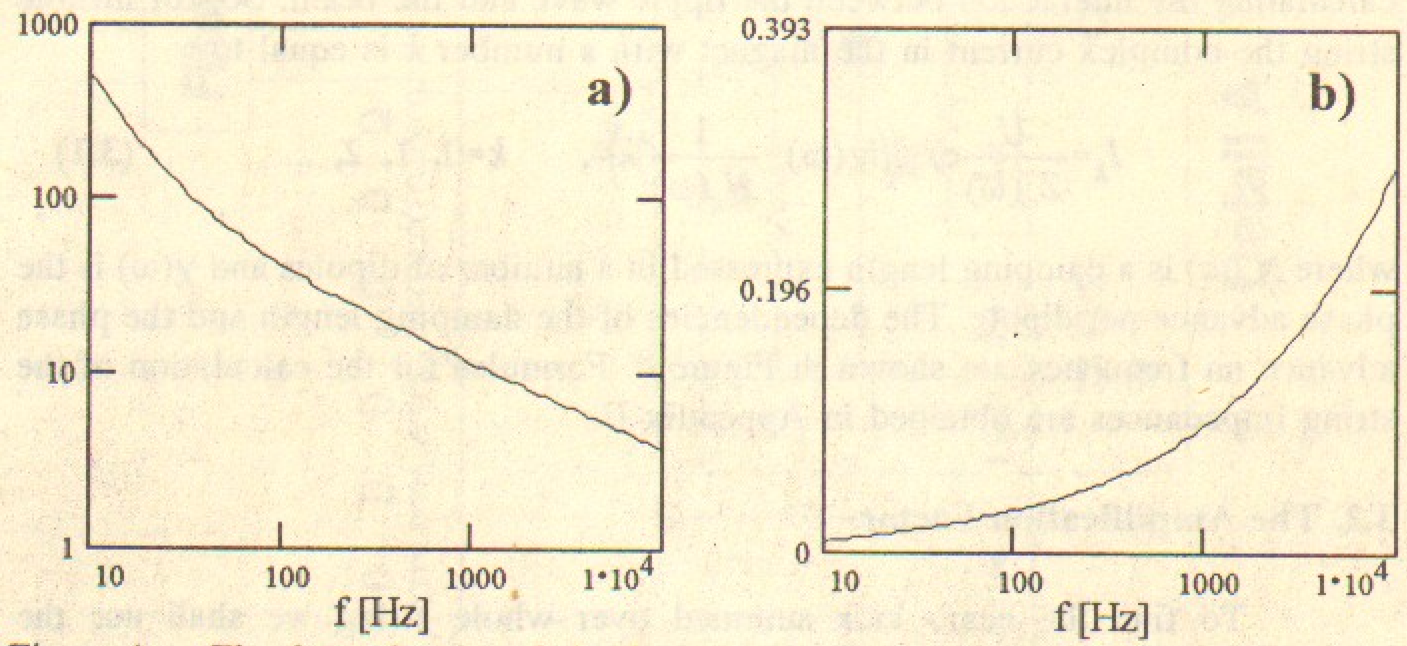

Figure 4. The dependencies of the damping length expressed in a number of dipoles *a)* and the phase advance per dipole *b)* on frequency; R_d=10 Ω.

$$\Delta Z = i\sum_{k=-\infty}^{\infty}\sqrt{\beta_k}\,\Delta\theta_k e^{i\mu_k}\,. \tag{3.2}$$

Here β_k is β-function in dipole the k, μ_k is the betatron phase advance between the drive point and the dipole k,

$$\Delta\theta_k = \theta_0 e^{-\delta_k}\cos(\phi_k - \omega t) \tag{3.3}$$

is a beam kick from dipole k, θ_0 is the kick value from the first dipole in the string, ϕ_k and δ_k are the phase advance and the damping coefficient of the wave propagation between the drive point and the dipole k. Substituting Eq.(3.3) into Eq.(3.2) one has

$$\Delta Z(t) = i\theta_0\sum_{k=-\infty}^{\infty}\sqrt{\beta_k}\,e^{i\mu_k-\delta_k}\cos(\omega t-\phi_k)\,. \tag{3.4}$$

For further analysis we need to take into account that there are two groups of dipoles. The magnets of the first group (marked by bold letters in Figure 2 are connected directly to the power supply while the other magnets are powered by return current. This magnets have a very small effect on the beam because the ripple wave is strongly damped before it reaches them. We shall neglect kicks of this dipoles. To simplify a solution we will neglect β-function variation in the lattice and symmetrize the electrical circuit (see Figure 5b). As was already said we also neglect the quadrupoles influence on the wave propagation.

To understand the influence of the dipole connection type we consider two cases of magnet connection. In the first case (case *a*) all magnets are connected serially. In the second case (case *b*) we take into account that the power buses change their places after each 5 dipoles as it was discussed. The electrical circuits for both cases are shown in Figure 5. One can see that the electrical circuit shown in Figure 5b is close to the real magnet disposal shown in Figure 2.

First, we consider the beam kick from differential mode. Take into account that the current wave propagates symmetrically, so that $\delta_k=\delta_{-k}$, $\phi_k=\phi_{-k}$, and for the betatron motion $\mu_k=-\mu_{-k}$, one obtains from Eq.(3.4):

$$\Delta Z_d(t) = 2i\theta_0\sqrt{\beta}\,\sum_{k=0}^{\infty}e^{-\delta_k}\cos(\omega t-\phi_k)\cos\mu_k\,. \tag{3.5}$$

The amplitude of the summed kick is equal to $|\Delta Z_d|_{max}/\beta^{1/2}$, thus for differential mode we finally have

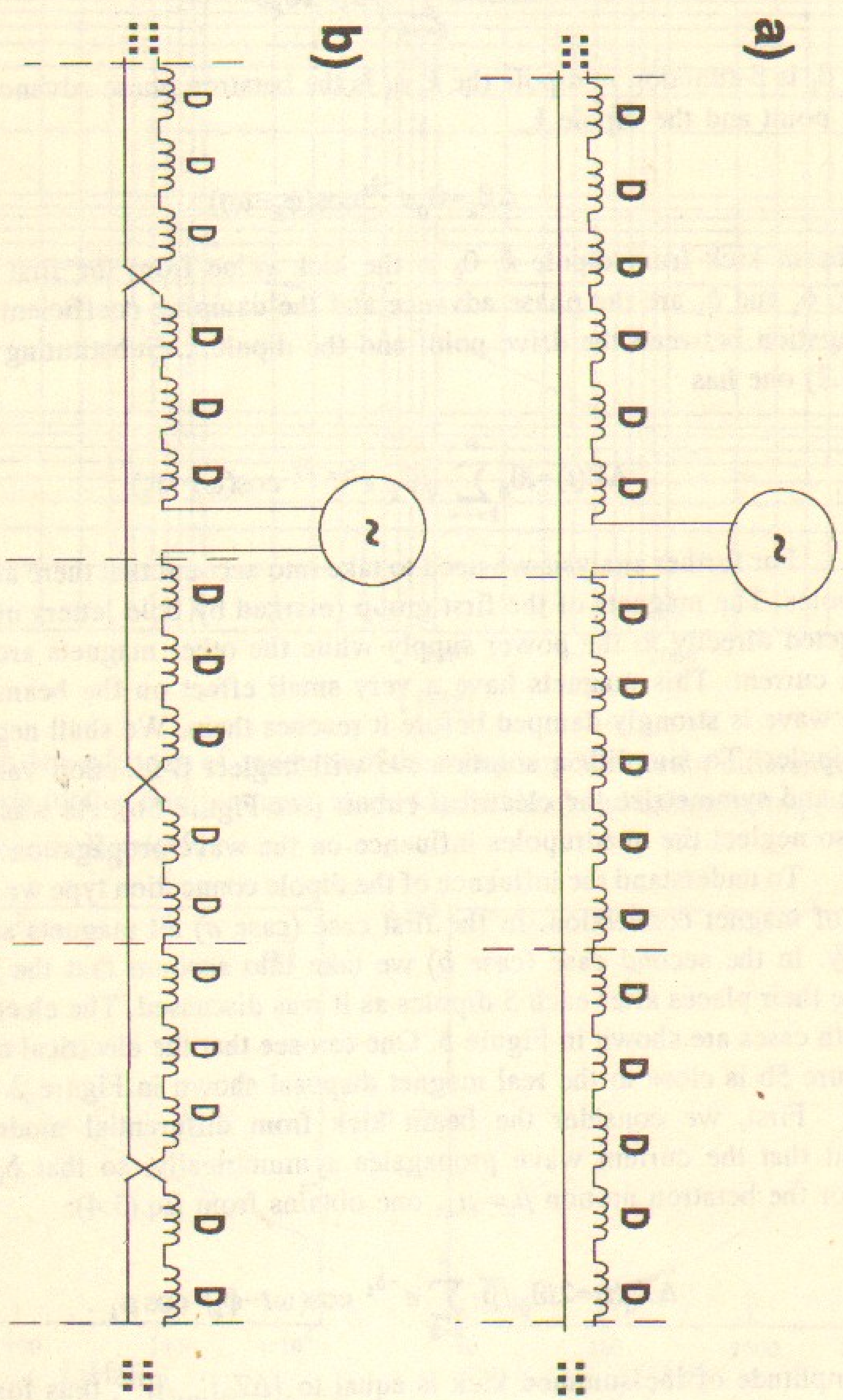

Figure 5. Simplified electrical circuit of the magnet string. a - simple scheme, b - scheme close to the real electrical circuit.

$$\theta_{e_d}=2\theta_0\ \Big|\sum_{k=0}^{\infty} e^{i\phi_k-\delta_k}\cos\mu_k\Big|\ . \tag{3.6}$$

Similarly, taking into account that $\delta_k=\delta_{-k}$, $\phi_k=\phi_{-k}+\pi$ for common mode the summed kick is equal to

$$\theta_{e_c}=2\theta_0\ \Big|\sum_{k=0}^{\infty} e^{i\phi_k-\delta_k}\sin\mu_k\Big|\ . \tag{3.7}$$

For the case *a*, when magnets are simply connected serially, we can write

$$\phi_k=k\chi(\omega)\ ,\qquad \delta_k=\frac{k}{N_d(\omega)}\ ,\qquad \mu_k=\mu k\ , \tag{3.8}$$

and sums (3.6) and (3.7) can be easily calculated. As a result one has

$$\theta_{a_d}=2\theta_0\ \Big|\sum_{k=0}^{\infty} e^{(i\chi-\frac{1}{N_d})k}\cos(\mu k)\Big| = \theta_0\left|\frac{1}{1-e^{i(\chi+\mu)-\frac{1}{N_d}}}+\frac{1}{1-e^{i(\chi-\mu)-\frac{1}{N_d}}}\right|, \tag{3.9}$$

for differential mode, and

$$\theta_{a_c}=2\theta_0\ \Big|\sum_{k=0}^{\infty} e^{(i\chi-\frac{1}{N_d})k}\sin(\mu k)\Big| = \theta_0\left|\frac{1}{1-e^{i(\chi+\mu)-\frac{1}{N_d}}}-\frac{1}{1-e^{i(\chi-\mu)-\frac{1}{N_d}}}\right|, \tag{3.10}$$

for common mode where μ is the betatron phase advance per dipole.

For the case *b* the power buses change their places with each 6-th dipole. As has been mentioned we shall neglect here the dipoles powered by return current (see Figure 5b). Although these dipoles do not kick the beam there is a betatron motion in them and, consequently, the beam gets an additional betatron phase advance in these magnets. In this case the summed angles will be equal to

$$\theta_{b_d}=2\theta_0\Big|\sum_{k=0}^{2} e^{(i\chi-\frac{1}{N_d})k}\cos\mu k+ \sum_{n=0}^{\infty}\sum_{k=0}^{4} e^{(i\chi-\frac{1}{N_d})(k+5n+3)}\cos(\mu\,(k+10n+8))\Big|, \tag{3.11}$$

for the differential mode, and

$$\theta_{b_c}=2\theta_0\Big|\sum_{k=0}^{2} e^{(i\chi-\frac{1}{N_d})k}\sin\mu k+ \sum_{n=0}^{\infty}\sum_{k=0}^{4} e^{(i\chi-\frac{1}{N_d})(k+5n+3)}\sin(\mu(k+10n+8))\Big|, \tag{3.12}$$

for the common mode.

The value θ/θ_0 is the summed (over the whole string) kick expressed in the kick value of the first magnet in the string. We shall call this value the amplification factor. The plots of the amplification factors (3.9)-(3.12) against the frequency for the SSC parameters are shown in Figure 6. As one can see the maximum value of the amplification factor is about 10 for the case *a* (ideal string) and ~2 times lower for the case *b* (real string). The value of the damping resistor has rather weak influence on the amplification factor.

3.3. The Summed Beam Kick

To get the kick angle for a given string voltage it is required to find the ripple of the magnetic field in the first dipole. In the considered above model of the dipole the magnetic field should be produced by an alternate current which flows through the inductances only. Thus, we need to introduce the field impedance Z_f of the infinite string, so that the ripple of magnetic field in the first dipole is

$$\frac{\Delta B}{B_0}=\frac{U}{Z_f I_0}, \tag{3.13}$$

where U is the ripple voltage on the string, B_0 and I_0 are the nominal magnetic field and current of the string. Dependance of Z_f on frequency is shown in Figure 7. Here we assumed that magnetic field of the dipole is equal to

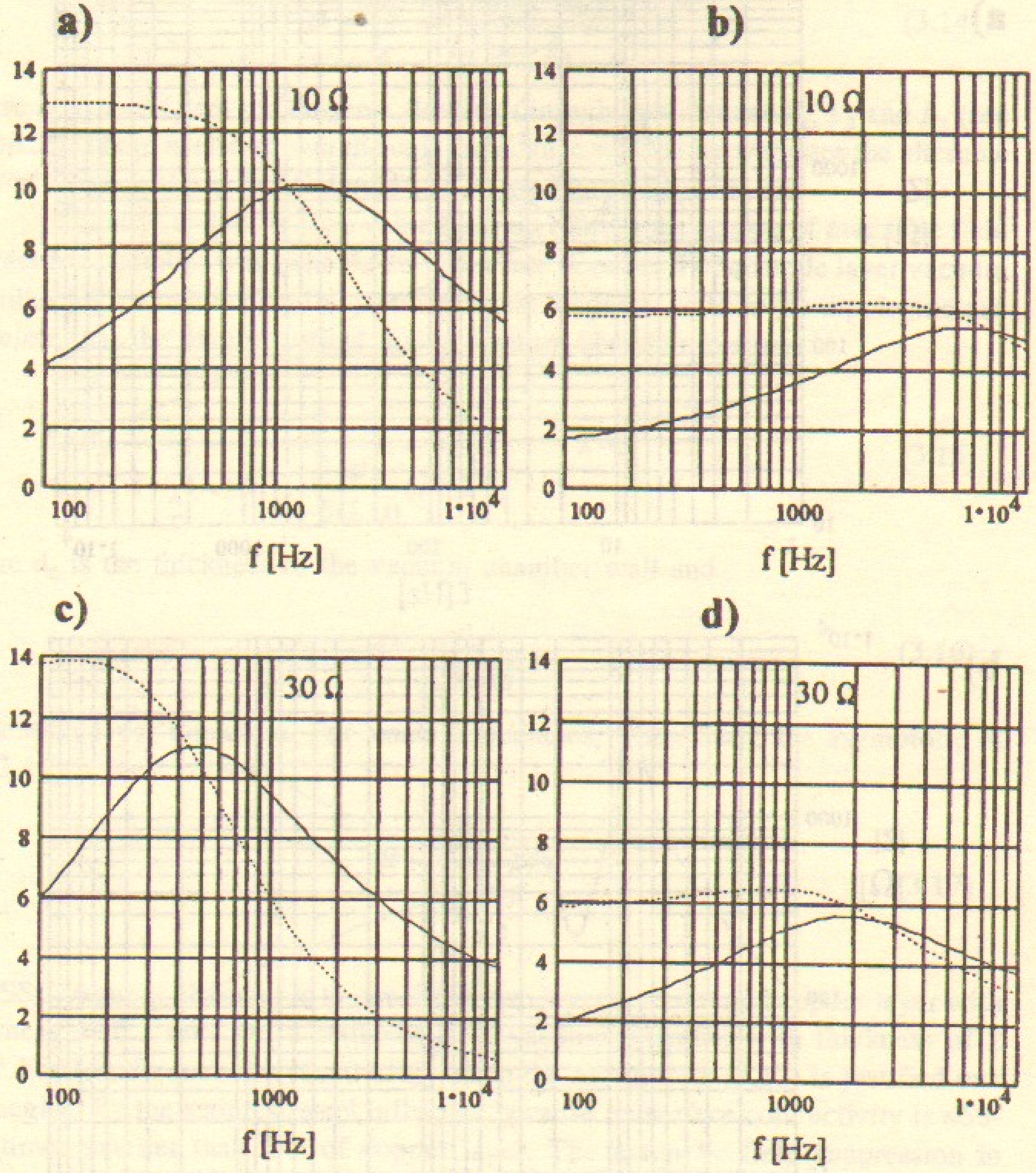

Figure 6. The dependences of the amplification factor on frequency for the simple electric circuit (case *a* - figures *a), c)*) and for the real electric circuit (case *b* - figures *b), d)*); solid line - differential mode, dashed line - common mode; figures *a), b)* - R_d=10 Ω; figures *c), d)* - R_d=30 Ω.

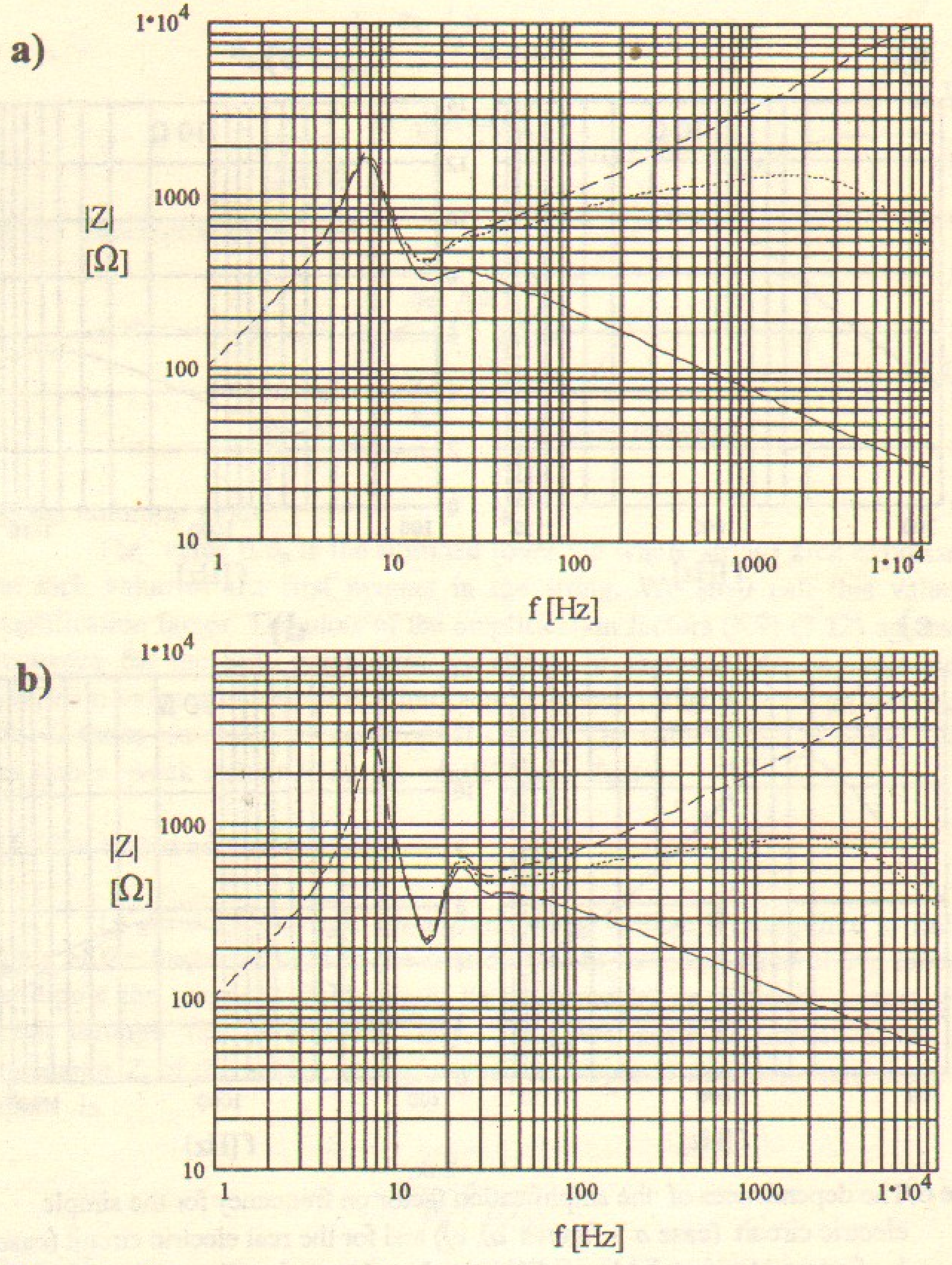

Figure 7. The dependencies of modules of the infinite string impedances on frequency; the full string impedance |Z| - solid line, the coil imredance $|Z_{coil}|$ - dotted line, the field impedance $|Z_{field}|$ - dashed line;
a) - R_d=10 Ω, *b)* R_d=30 Ω.

$$B_{-}=\frac{B_0}{I_0}\,\frac{L_1I_1+L_2I_2+L_3I_3}{L_1+L_2+L_3}\,, \tag{3.14}$$

where I_1, I_2 and I_3 are the currents flowing through inductances L_1, L_2 and L_3 (see Figure 2). There is also shown the coil impedance which characterizes the alternate current flowing through the dipole itself (see Figure 2).

Finally, to get the kick value the skin effect suppression of magnetic field by vacuum chamber walls should be taken into account. For a single layer vacuum chamber with a radius *a* and uniform external magnetic field in the dipole one can calculate that the magnetic field inside vacuum chamber is equal to

$$B=B_0\,\frac{2}{1+\frac{1}{2}(\lambda+\lambda^{-1})-\frac{a}{\delta(1-i)}(\lambda-\lambda^{-1})}\,,\qquad \lambda=e^{(1+i)\frac{d_0}{\delta}}\,, \tag{3.15}$$

where d_0 is the thickness of the vacuum chamber wall and

$$\delta=\frac{c}{\sqrt{2\pi\sigma\omega}}\,, \tag{3.16}$$

is the skin layer thickness. For small frequencies, when $\delta \gg d_0$, the asymptotic of Eq.(3.15) is well known

$$B=\frac{B_0}{1+i\frac{ad_0}{\delta^2}}\,, \tag{3.17}$$

The SSC vacuum chamber is to consist of two layers: an internal copper layer with thickness of 0.1 mm and a stainless steel vacuum chamber with thickness of 2 mm.[1] For low frequencies ($f \leq 10$ kHz) when the asymptotic (3.17) is justified one can neglect by the stainless steel influence because its surface conductivity is ≈50-100 times smaller than that of copper layer. The magnetic field suppression in accordance with (3.17) is shown in Figure 8. For frequencies higher than 10 kHz

[1] *We consider here the worst case when the internal liner is not used. In the case with the liner the value of the magnetic field suppression will be higher.*

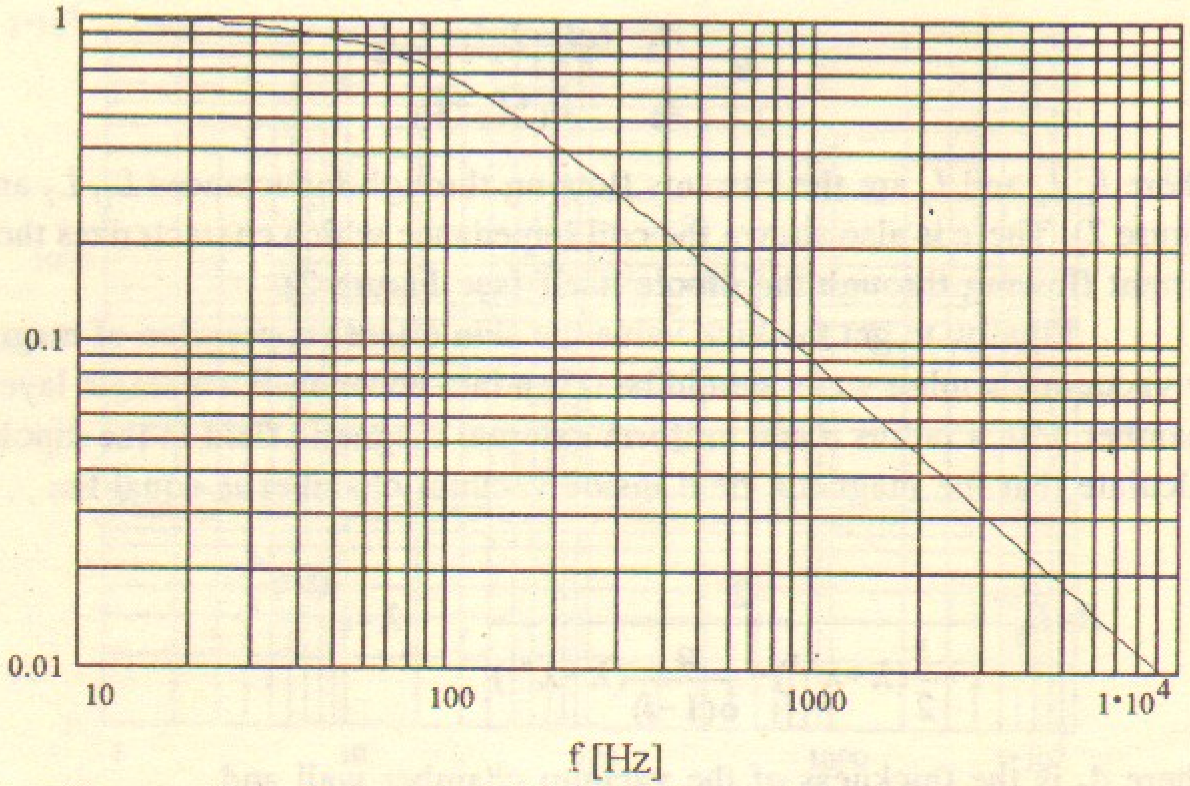

Figure 8. Suppression of magnetic field ripple due to the skin effect in vacuum chamber walls; the vacuum chamber radius - 17 mm, the thickness of copper layer - 0.1 mm, RRR=30 (temperature of copper 4 K^0).

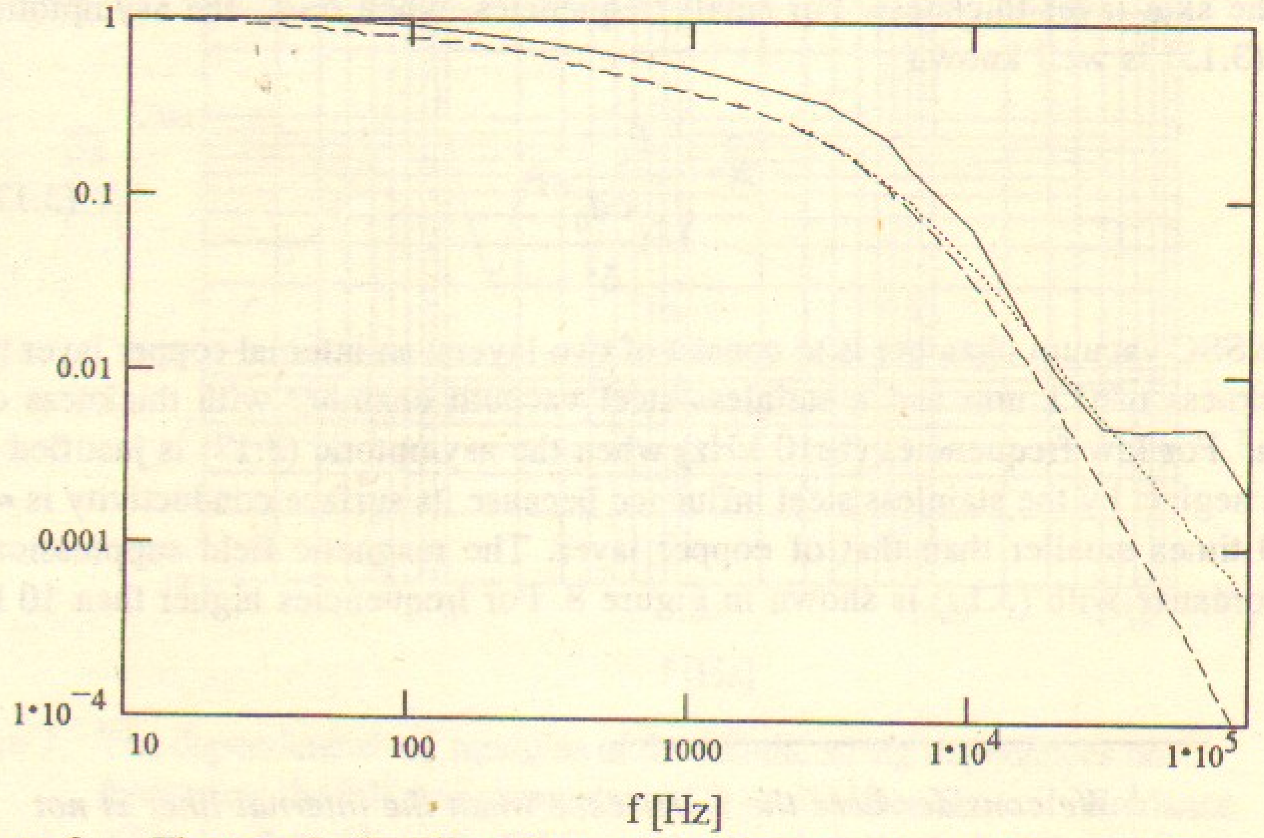

Figure 9. The transfer functions from coil current to magnetic field ; solid line - the experimental measurements[3], dotted line - without skin effect suppression, dashed line - with skin effect suppression.

the skin layer thickness in stainless steel becomes comparable with the wall thickness, then the stainless steel conductivity should be taken into account that yields an additional suppression of magnetic field in comparison with Eq.(3.17).

The comparison of the experimentally measured suppression of the magnetic field ripple[6] for the dipole at room temperature with predictions of the model are shown in Figure 9. One can see a satisfactory agreement for frequencies lower than 30 kHz. For higher frequencies the developed dipole model is not applicable.

The dependencies of the summed kick values on frequency for different values of damping resistors and string voltage[2] of 1 V are shown in Figure 10. One can see that the summed kick for the damping resistors of 10 Ω is ≈2 times smaller than for 30 Ω resistors.

3.4. The Amplification Factor for Quadrupole Kicks

The dipoles and quadrupoles are connected serially in the collider arcs as shown in Figure 2. Thus, the current ripple in quads is the same as in dipoles. Without the feedback system due to a very small beam size the ripple in quads produces much smaller emittance growth than ripple in dipoles. But the use of a powerful feedback system allows to suppress strongly the contribution of the dipole kicks while the contribution of the quadrupole kicks cannot be suppressed (see section 2.4).

To find the emittance growth rate it is necessary to take into account that kicks of different quads are correlated. For the electric circuit shown in Figure 2 the effective value of quadrupole kick for the differential mode is

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[2] *According to the definition given above it means that for the differential mode the voltages on input buses are ±1 V with respect to the ground and have opposite polarities. For the common mode the voltages are 1 V with respect to the ground and have the same polarity for both buses.*

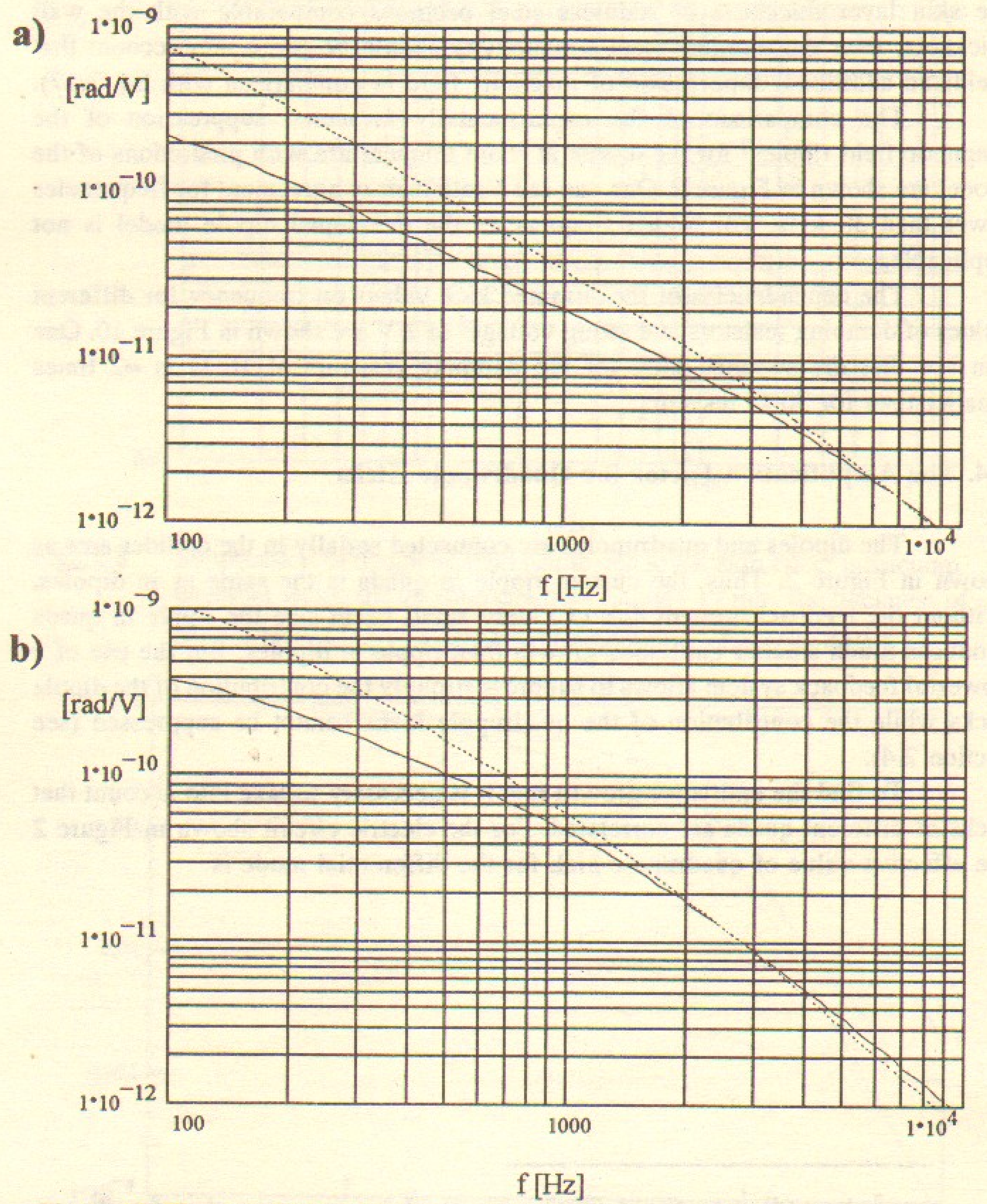

Figure 10. The transfer function from voltage of power supply ripple to the summed beam kick of one circuit of 480 dipoles; differential mode - solid line, common mode - dotted line; *a)* - R_d=10 Ω, *b)* R_d=30 Ω.

$$\Delta_F=\Delta_{F0}\sum_{n=0}^{\infty} e^{10(i\chi(\omega)-\frac{1}{N_d(\omega)})k}+e^{10(i\chi(\omega)-\frac{1}{N_d(\omega)})(k+1)}=$$

$$=\Delta_{F0}\frac{1+e^{10(i\chi(\omega)-\frac{1}{N_d(\omega)})}}{1-e^{10(i\chi(\omega)-\frac{1}{N_d(\omega)})}}. \qquad (3.18)$$

Here $N_d(\omega)$ is the damping length expressed in number of dipoles and $\chi(\omega)$ is the phase advance per dipole (see Eq.(3.1)), $\Delta_{F0}=\Delta G/G$ is a relative amplitude of gradient perturbation in the first quad, and we took into account that an active quadrupole occurs after each 10 dipoles. The first and the second addends in Eq.(3.18) determine contributions of the left- and right-hand branches of the string, respectively.

Let it be, the same as for the dipoles, the damping resistors R_d are connected across the quads in parallel with their impedance. Then the current ripple through quad is

$$I=\frac{U}{Z_\infty}\frac{R_d}{R_d+i\omega L_q}, \qquad (3.19)$$

where L_q is the quadrupole inductance and U is the string voltage. Unlike in the dipole case we neglect here the complications in quad circuit because its inductance is much smaller.

Taking into account the quadrupole magnetic field suppression due to skin effect (compare (3.17))

$$B=\frac{B_0}{1+2i\frac{ad_0}{\delta^2}}, \qquad (3.20)$$

we finally have the relative perturbation of the quadrupole field summed over all quads of the string

$$\Delta_F=\frac{U}{I_0 Z_\infty}\frac{R_d}{R_d+i\omega L_q}\frac{1}{1+2i\frac{ad_0}{\delta^2}}\frac{1+e^{10(i\chi(\omega)-\frac{1}{N_d(\omega)})}}{1-e^{10(i\chi(\omega)-\frac{1}{N_d(\omega)})}}, \qquad (3.21)$$

where I_0 is the nominal string current (d.c.). Substituting Eq.(3.21) into Eq.(2.23) one can get the emittance growth rate due to the resonance harmonic perturbation of power supply voltage.

To get the amplification factor for the common mode one should replace the sign "plus" between addends in the first part of Eq.(3.18) with "minus". As a result one has the amplification factor for the common mode equal to 1 and the last factor in Eq.(3.12) should be replaced by 1.

4. The Collider Tune and the Transverse Emittance Growth Rate due to Power Supply Ripple and Noise

In the general case a perturbation of power supply current should consist of two parts: a random noise and a ripple on harmonics of the power line. For the design of power supplies[7] under consideration with a passive suppression of the voltage ripple the main part of perturbation should be due to the ripple. As follows from previous sections the emittance growth due to the ripple will be equal to zero if no harmonic has an overlapping with the betatron frequency and its sidebands. The condition of overlapping is that a frequency of one of power line harmonic (n·60 Hz) should be inside one of betatron resonance bands

$[\,|\nu-k|\,f_0,\ \ |\nu-k+\xi|\,f_0]$ - for the dipole perturbation and

$[\,|2\nu-k|\,f_0,\ \ |2(\nu+\xi)-k|\,f_0]$ - for the quadrupole perturbation.

Unlike to the Tevatron and the SppS for the SSC the betatron tune spread $\xi f_0 \approx 12.5$ Hz is smaller than the distance between harmonics (60 Hz - for the dipole case and 30 Hz - for the quadrupole case). It allows one to choose the collider tune between the power line harmonics at least for the lowest ones which have larger amplitudes. Unfortunately, one cannot avoid an overlapping at higher harmonics because the revolution frequency is not a harmonic of the power line frequency. The diagram of mode overlapping for the collider is shown in Figure 11. Here the fractional part of the unperturbed tune $f=[\nu]f_0$ is put on abscissa axis. The harmonic number and its frequency are put on the ordinate axis. Horizontal bars show the regions where the overlapping occurs with a given harmonic of the power line. For example, for a fractional part of the betatron tune $[\nu]=0.4635$ ($f=f_0{*}[\nu]=1595$ Hz) we have an overlapping with a harmonic of 203 (frequency 12180 Hz) which produces a dipole perturbation and a harmonic of 53 (frequency 3180 Hz) which produces a quadrupole perturbation. So although we cannot absolutely avoid the mode overlapping we can choose the collider tune at which the overlapping occurs at high frequencies only.

Contribution to the emittance growth time $(1/\varepsilon\cdot d\varepsilon/dt)^{-1}$ due to the ripple of the one from ten power supplies are shown in Figure 12. The effects of dipoles

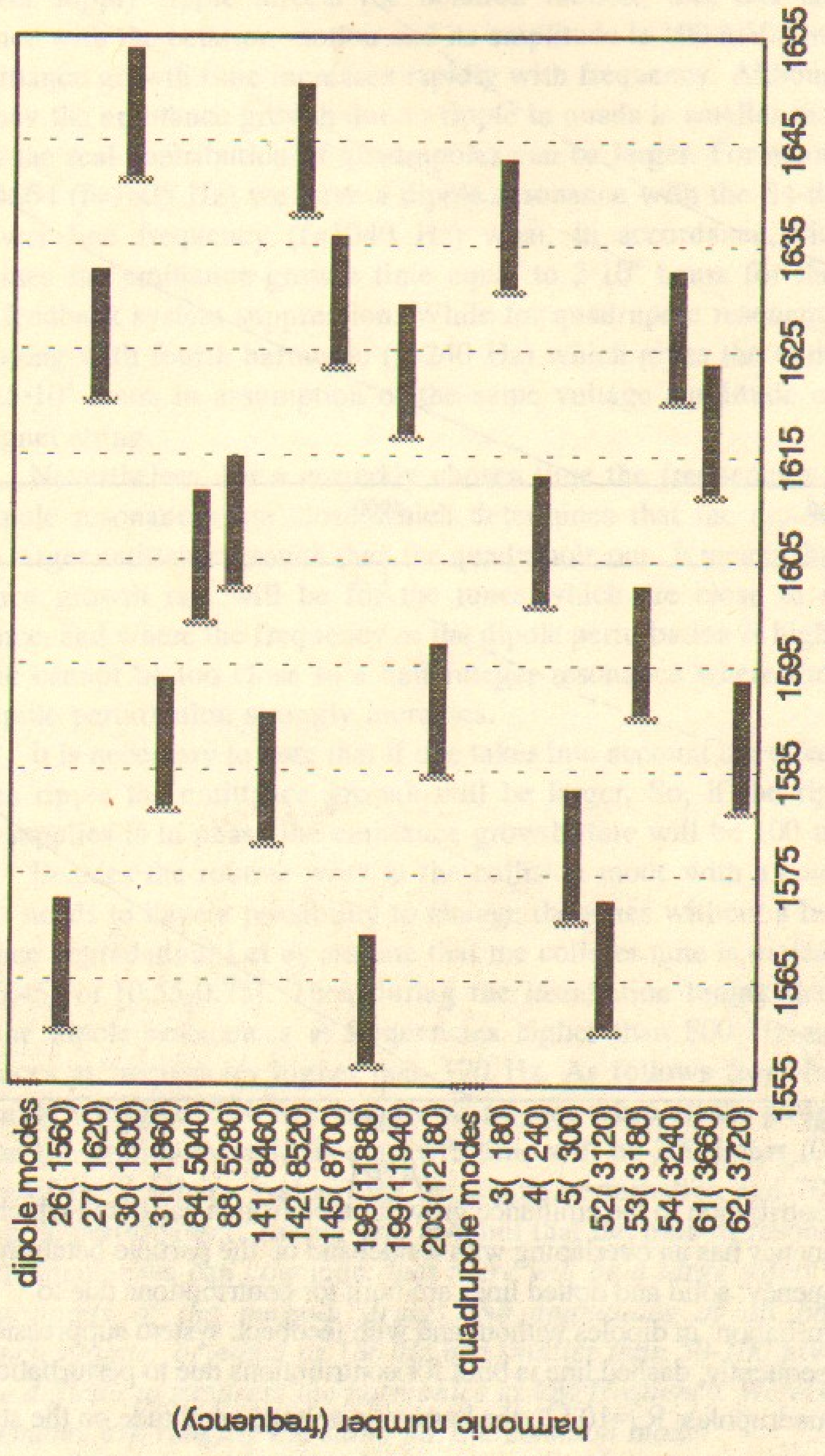

Figure 11. Resonance sidebands from ripple of the power supply for dipole and quadrupole modes.

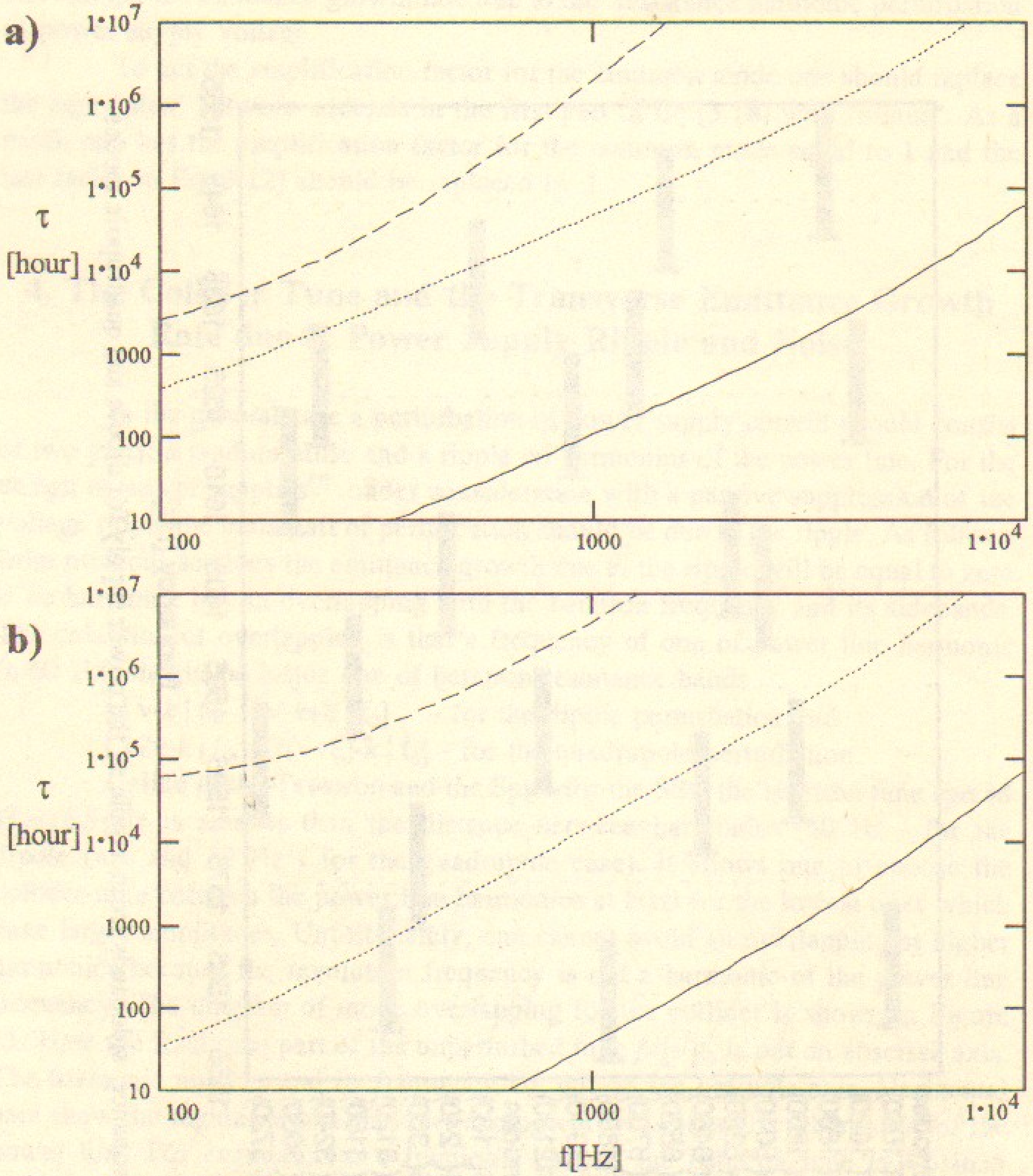

Figure 12. A contribution to the emittance growth time τ from a harmonic which frequency has an overlaping with a sideband of the particle betatron frequency: solid and dotted lines are built for contributions due to perturbation in dipoles without and with feedback system suppression, consequently; dashed line is built for contributions due to perturbation in quadrupoles; R_d=10 Ω, the harmonic voltage amplitude on the string is 100 mV. Plots a) and b) are built for the differential and common modes, consequently.

and quadrupoles are separated on this plot. It is suggested that only one harmonic of power supply ripple affects the betatron motion; that this harmonic is in resonance with the betatron motion and its amplitude is 100 mV. One can see that the emittance growth time increases rapidly with frequency. Although for a fixed frequency the emittance growth due to ripple in quads is smaller than that for the dipoles the real contribution of quadrupoles can be larger. For example at a tune [ν]=0.4664 (f=1605 Hz) we have a dipole resonance with the 84-th harmonic of the power line frequency (f=5040 Hz) what, in accordance with Figure 12, determines the emittance growth time equal to $3 \cdot 10^6$ hours for the case with a strong feedback system suppression. While for quadrupole resonance we have an overlapping with fourth harmonic (f=240 Hz) which gives the emittance growth time $1.5 \cdot 10^4$ hours in assumption of the same voltage amplitude of 100 mV on the magnet string.

Nevertheless, for a correctly chosen tune the frequencies of dipole and quadrupole resonances are close which determines that the dipole perturbation gives a larger emittance growth than the quadrupole one. It means that the minimal emittance growth rate will be for the tunes which are close to a half integer resonance, and where the frequency of the dipole perturbation is higher. Of course, the tune cannot be too close to a half integer resonance where the effect of the quadrupole perturbation strongly increases.

It is necessary to note that if one takes into account the effect of all power supplies ripple the emittance growth will be larger. So, if the ripple of all 10 power supplies is in phase the emittance growth time will be 100 times smaller.

Besides the routine work at the collision mode with a constant tune the collider needs to have a possibility to change the tunes without a beam loss or an emittance degradation. Let us assume that the collider tune is varied in the region [0.25-0.45] or [0.55-0.75]. Then during the installation tuning the beam should cross the dipole resonances at frequencies higher than 800 Hz and quadrupole resonances at frequencies higher than 170 Hz. As follows from Figure 12 for a differential voltage of 100 mV and the worst case when all the power supplies are in phase the minimum emittance growth time will be 300 hours (0.6 hour in the absence of the feedback system) for the dipole perturbation and 70 hours for the quadrupole perturbation. Taking into account that the time of resonance crossings is much smaller we can conclude: *that there will be a large safety room if at the feeding points of the magnet strings the amplitudes of all harmonics with frequencies higher or equal to 180 Hz are smaller than 50-100 mV. Of course, it is more difficult to suppress the harmonics at low frequency. We also can see that requirements are roughly the same for the common mode.*

For random noise the string voltage requirements are much softer. As follows from Eq.(2.1) and Figure 10 the spectral density of noise $\sum S(\omega_n)$ has to be smaller than $5\cdot10^{-4}$ V^2/Hz. For the white noise in a frequency band of 3.4 kHz it gives r.m.s. string voltage of about 1 V.

5.Longitudinal Emittance Growth and Limitation of the Noise Spectral Density at Synchrotron Frequency

As was shown in Section 1 (see also Ref.[1],[2]) the low frequency perturbation does not produce a transverse emittance growth. Nevertheless, the transfer function from the string voltage to the magnetic field increases fast with frequency decrease. So, even a small low frequency voltage perturbation should produce a large perturbation of the orbit length, which can cause a longitudinal emittance growth.

As was shown in ref.[8] the growth rate of the r.m.s. phase size of a bunch is equal to

$$\frac{d}{dt}\langle\phi^2\rangle=(\pi\omega_0 q)^2 S_L(\Omega_s) \ , \tag{5.1}$$

where α is the momentum compaction factor, q is the harmonic number of the RF voltage, ω_0 and Ω_s are the revolution and synchrotron frequencies and $S_L(\Omega_s)$ is the spectral density of relative length perturbation normalized to

$$\left\langle\frac{\Delta L^2}{L^2}\right\rangle=\int_{-\infty}^{\infty} S_L(\omega)d\omega \ . \tag{5.2}$$

From Eq.(5.1) one can see that as in the case of the transverse motion the resonance frequency only produces an emittance growth[3]. The synchrotron frequency of the collider is changed from 8 Hz at injection to 4 Hz at the top energy. So the noise of the power supply at these frequencies should be dangerous.

The relative length change of the closed orbit is equal to[8]

[3] *We neglect by sidebands $\omega_n=\Omega_s+n\omega_0$, $n=\pm1, \pm2,...$ because their frequencies are three order of magnitude larger then synchrotron frequency.*

$$\frac{\Delta L}{L}=\frac{1}{2\pi R}\sum_{n=1}^{N}\psi_n\Delta\theta_n \ , \tag{5.3}$$

where θ_n is the angle kick associated with perturbation of magnetic field in the magnet n, ψ_n is the dispersion function at the magnet n and R is the average storage ring radius. For the case of very small or zero frequencies, when $\Delta\theta_n/\theta_n=\Delta I/I$, Eq.(5.3) is transformed to a well known expression

$$\frac{\Delta L}{L}=\alpha\frac{\Delta B}{B}=\alpha\frac{\Delta I}{I} \ , \tag{5.4}$$

where $\Delta B/B$ and $\Delta I/I$ are relative perturbations of the magnetic field and current in dipoles.

One can see in Figure 3 that both common and differential modes have the resonance behavior in vicinity of the synchrotron frequency. Distributions of voltage and current amplitudes in the string for the differential mode are shown in Figure 13. One can see that the wave damping in the synchrotron frequency range is small, and due to a resonance the current amplitude at the string center is even larger than at the string ends.

As can be seen from Figure 7 for frequencies smaller than 10 Hz we can neglect the magnetic field suppression by vacuum chamber walls and current redistribution in the coil, so a full string current produces the magnetic field. In this case the kick of the magnet n is

$$\frac{\Delta\theta_n}{\theta_n}=\frac{\Delta I_n}{I}=\frac{U_{n+1}-U_n}{Z_d I} \ , \tag{5.5}$$

where U_n and U_{n-1} are the voltages on the input and output of the dipole n, $Z_d=i\omega L_d$ is the impedance of one dipole and L_d is its inductance. Substituting Eq.(5.5) into Eq.(5.3), averaging over one lattice cell and summing up one can get

$$\frac{\Delta L}{L}=\frac{1}{2\pi R}\sum_{n=1}^{N}\psi_n\theta_n\frac{U_{n+1}-U_n}{Z_d I}=\frac{\langle\psi\rangle}{R}\frac{U_N-U_0}{NZ_d I}=\alpha\frac{U_N-U_0}{NZ_d I} \ , \tag{5.6}$$

where $\langle\psi\rangle$ is the dispersion function averaged over a lattice cell, U_N-U_0 is the full string voltage and $\alpha=\langle\psi\rangle/R$ is the momentum compaction factor. As one can see the expression for the orbit lengthening (5.6) is just the same as in the case of very low frequencies where we can neglect the wave propagation in the string. Substituting Eq.(5.6) into Eq.(5.1) we finally have the growth rate of the r.m.s. phase size of bunch to be equal to

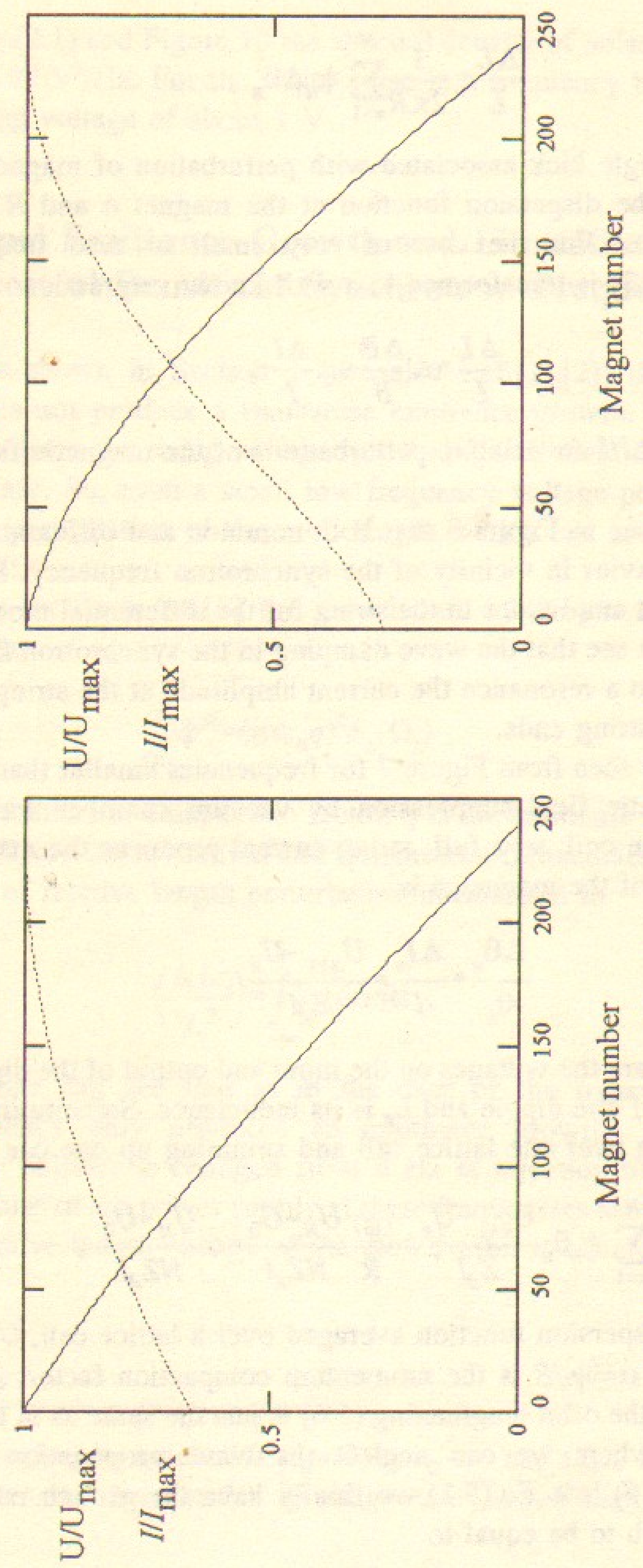

Figure 13. Distributions of voltage (solid line) and current (dotted line) amplitudes along magnet string; R_d=10 Ω.

$$\frac{d}{dt}\langle\phi^2\rangle=\left(\frac{\pi\,\omega_0 q\alpha}{N\omega L_d I}\right)^2 S_U(\Omega_s)\ , \qquad (5.7)$$

where $S_U(\omega)$ is the spectral density of string voltage.

For the Collider α=9.1·10^{-5}, L_d=65.6 mH, $\omega_0 q/2\pi$=360 MHz, I=6500 A, the bunch length σ_s=6 *cm* which for RF frequency of 360 MHz determines the r.m.s. phase size of the bunch $\langle\phi^2\rangle^{1/2}\approx0.45$. Supposing that the noises of all 10 power supplies are independent one obtains that the longitudinal emittance growth time $\tau=\langle\phi^2\rangle/(d\langle\phi^2\rangle/dt)$ from 10 power supplies will be 10 times smaller than for one. For the growth time equal to 30 hours (that is five times larger than the SR damping time) one obtains from Eq.(5.7) the limitation on the spectral density of the string voltage, $S_U\leq10^{-4}$ V^2·s in a frequency band of [4-8] Hz. It means that the noise voltage in this frequency band has to be smaller than 70 mV.

6.Discussion

The main conclusion that follows from this work is that the transverse and longitudinal emittances grow only under the influence of resonance frequencies, i.e. the frequencies which are inside the betatron or synchrotron frequency spread and their sidebands: $f_n=f_0(\nu-n)$ for the betatron motion or $f=f_s$ for the synchrotron one.

In the betatron frequency range $f\geq$180 Hz the spectral power of the power supply voltage should consist of two parts. The first one consists in narrow peaks at power line harmonics $f_k=k$·60 Hz which make the main contribution to the voltage perturbation. The second one consists in the plato of random noise which originates from the noise in the power line voltage and from the internal noises of magnets[4]. Fortunately the betatron tune spread (~12 Hz for 4 IPs) is smaller than the power line frequency (60 Hz). It allows one to avoid an overlapping of betatron and power supply frequencies up to at least 10 kHz. It facilitates the requirements to power supplies because the overlapping can occur only during a small time of the resonance crossing. The important role is played by the transverse feedback system which can suppress the emittance growth time by three orders of magnitude.

In the synchrotron frequency range (4-8 Hz) a noise in the power supply

[4]*Due to helium flow, its evaporation, et.c.*

voltage should produce a longitudinal emittance growth. Similar as for the betatron motion the longitudinal emittance growth can be suppressed by the longitudinal feedback system but a value of this suppression will be much smaller because of a large synchrotron tune spread.

The main requirements to the power supplies of collider bending magnets (for each of ten power supplies) are:

1. The amplitude of any power line harmonic with frequency equal to or larger than 180 Hz should be smaller than 100 mV.
2. The spectral density for frequencies larger 180 Hz should be smaller than $5\cdot10^{-4}$ V^2/Hz. For the white noise in the frequency band of 3.4 kHz it gives r.m.s. string voltage about of 1 V.
3. The spectral density in the frequency band of the synchrotron motion (4-8 Hz) should be smaller than 10^{-3} V^2/Hz. It means that the noise voltage in this frequency band has to be smaller than 70 mV.

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Appendix A
Dependence of Betatron Tune Shift on Amplitude due to Beam-Beam Effects and Distribution Function over Particle Tune

For a round beam the kick values of a particle with coordinates x,y at the IP are equal to

$$\delta p'_x=\frac{8\pi\xi x}{r^2}\left(1-\exp\left(-\frac{r^2}{2}\right)\right), \qquad x, p_x \leftrightarrow y, p_y . \tag{A1}$$

We use here the same coordinates as in Section 1. Taking into account that in the first order of the perturbation theory the tune shifts are

$$\Delta\nu_x=\frac{1}{2\pi a_x}\langle\delta p_x\sin(\phi_x)\rangle , \qquad \Delta\nu_y=\frac{1}{2\pi a_y}\langle\delta p_y\sin(\phi_y)\rangle , \tag{A2}$$

we can express tune shift by an integral[9]:

$$\Delta\nu_x(a_x,a_y)=\frac{\xi}{\pi^2}\int_0^{2\pi}d\psi_x\int_0^{2\pi}d\psi_y\frac{s_x^2}{a_x^2s_x^2+a_y^2s_y^2}\left(1-\exp\left(-\frac{a_x^2s_x^2+a_y^2s_y^2}{2}\right)\right), \qquad x\leftrightarrow y, \tag{A3}$$

where a_x,a_y are the amplitudes of oscillations and ϕ_x, ϕ_y are their phases. We evaluated the integral numerically. The results of the calculations for horizontal tune shift are shown in Figure 14.

To calculate the mean value $\langle\Delta\nu\rangle$ and the mean square value $\langle\Delta\nu^2\rangle$ of the tune shift one has to integrate the tune shift with the distribution function

$$\langle\Delta\nu\rangle=\langle\Delta\nu\rangle_x=\langle\Delta\nu\rangle_y=\frac{1}{4\pi}\int_{-\infty}^{\infty}da_x\int_{-\infty}^{\infty}da_y\ \Delta\nu_x(a_x,a_y)\exp\left(-\frac{a_x^2s_x^2+a_y^2s_y^2}{4}\right)$$

$$\langle\Delta\nu^2\rangle=\langle\Delta\nu^2\rangle_x=\langle\Delta\nu^2\rangle_y=\frac{1}{4\pi}\int_{-\infty}^{\infty}da_x\int_{-\infty}^{\infty}da_y\ \Delta\nu_x^2(a_x,a_y)\exp\left(-\frac{a_x^2s_x^2+a_y^2s_y^2}{4}\right) \tag{A4}$$

Here we took into account that because of oscillations, $2\langle x^2\rangle=\langle a_x^2\rangle$. To calculate the dispersion from (A4) one has to use the general equation

$$\sqrt{\delta\nu^2}=\sqrt{\langle\Delta\nu^2\rangle-\langle\Delta\nu\rangle^2} . \tag{A5}$$

The numerical calculations determine the following values

$$\sqrt{\overline{\delta\nu^2}}/\xi = 0.21\pm0.001\ , \qquad \text{(A6)}$$

$$\langle\Delta\nu\rangle/\xi = 0.67\pm0.003\ .$$

To build the distribution function over betatron frequency the next expression was used

$$f_\nu(\nu_x)=\frac{1}{4\pi}\int_{-\infty}^{\infty} da_x \int_{-\infty}^{\infty} da_y\ \ \delta\big(\nu-\Delta\nu_x(a_x,a_y)\big)\ \exp\left(-\frac{a_x^2 s_x^2+a_y^2 s_y^2}{4}\right), \qquad \text{(A7)}$$

$$x \leftrightarrow y\ .$$

Figure 15 shows the results of numerical calculations of this distribution function. One can see that the maximum distribution function is about $1.75/\xi$.

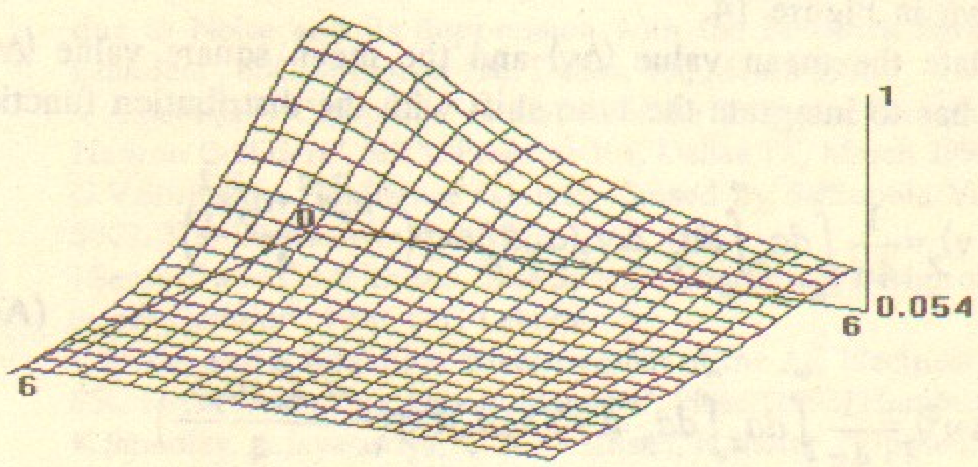

Figure 14. Dependence of betatron tune shift $\delta\nu_x/\xi$ on oscillation amplitudes.

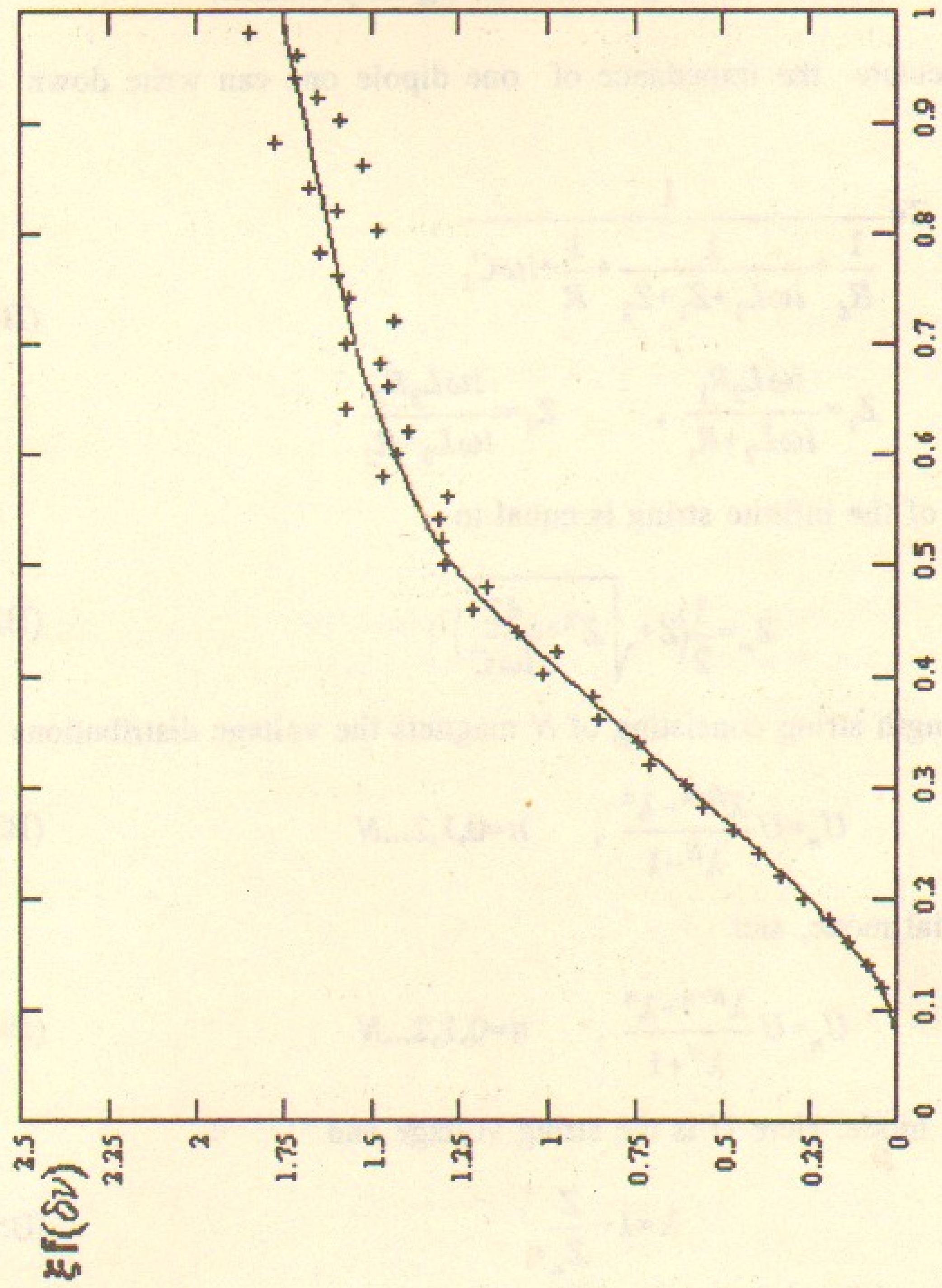

Figure 15. The distribution function over betatron tune

Appendix B
The Calculation of the String Impedances

To calculate the impedance of one dipole one can write down (see Figure 2)

$$Z=\frac{1}{\frac{1}{R_d}+\frac{1}{i\omega L_1+Z_1+Z_2}+\frac{1}{R}+i\omega C_1}\,, \tag{B1}$$

$$Z_1=\frac{i\omega L_2R_1}{i\omega L_2+R_1}\,,\qquad Z_2=\frac{i\omega L_3R_2}{i\omega L_3+R_2}\,.$$

The impedance of the infinite string is equal to

$$Z_\infty=\frac{1}{2}\left(Z+\sqrt{Z^2+\frac{4Z}{i\omega C}}\right)\,. \tag{B2}$$

For the finite length string consisting of N magnets the voltage distributions are

$$U_n=U\frac{\lambda^{N-n}-\lambda^n}{\lambda^N-1}\,,\qquad n=0,1,2...N \tag{B3}$$

for the differential mode, and

$$U_n=U\frac{\lambda^{N-n}+\lambda^n}{\lambda^N+1}\,,\qquad n=0,1,2...N \tag{B4}$$

for the common mode. Here U is the string voltage and

$$\lambda=1-\frac{Z}{Z_\infty} \tag{B5}$$

is the eigen number. The impedances are

$$Z_d=2Z\frac{\lambda^N-1}{\lambda^N-1-\lambda^{N-1}+\lambda} \tag{B6}$$

for the differential mode, and

$$Z_c=Z\frac{\lambda^N+1}{\lambda^N+1-\lambda^{N-1}-\lambda} \tag{B7}$$

for the common mode.