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VACUUM POLARIZATION CORRECTION TO THE POSITRONIUM DECAY RATE

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НОВОСИБИРСК

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to the Positronium Decay Rate

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ABSTRACT

Correction $\sim \alpha^2$ to the positronium decay rate, induced by one-loop vacuum polarization diagram, are calculated. Their relative values are $0.4468(3)(\alpha/\pi)^2$ for para- and $0.960(3)(\alpha/\pi)^2$ for orthopositronium.

The measured value of orthopositronium decay rate is [1]

$$\Gamma_{exp} = 7.0482(16)\,\mu s^{-1}. \tag{1}$$

At the same time, its theoretical value (including corrections $\sim (\alpha/\pi)$ and $\sim \alpha^2 \log \alpha$) is firmly established [2–7] and constitutes

$$F_{1th} = m\alpha^6\,\frac{2(\pi^2-9)}{9\pi}\left[1 - 10.28\,\frac{\alpha}{\pi} - \frac{1}{3}\,\alpha^2 \log\frac{1}{\alpha}\right] = 7.03830\,\mu s^{-1}. \tag{2}$$

To lift the disagreement between them by $(\alpha/\pi)^2$–corrections, the factor at $(\alpha/\pi)^2$ should be 250(40), which may look unreasonably large and very unusual for QED. To remove this discrepancy would be of importance for clear understanding of the relativistic bound-state problem in QED.

However, there exists at least one contribution of the second order with a large coefficient at $(\alpha/\pi)^2$; it arises when we square the one-loop correction to the annihilation amplitude. It is equal to [8]

$$\Gamma_{2th} = 28.8(2)(\alpha/\pi)^2\Gamma_0 \tag{3}$$

(here Γ_0 is the lowest-order decay rate). Calculation of two-loop correction to the annihilation amplitude is required to obtain the total correction $\sim \alpha^2$ to the orthopositronium decay rate. There exists an opinion [9] that numerical factor at $(\alpha/\pi)^2$ in the amplitude can also be large.

There is a large number of diagrams which contribute to the second-order amplitude, but only sum of them is gauge invariant. Nevertheless, there is a class of the two-loop diagrams sum of which is gauge invariant by itself. It consists of diagrams which contain photon self-energy insertion (these diagrams for orthopositronium (o-Ps) and parapositronium (p-Ps) are displayed in Fig.2 and Fig.1 respectively). In this paper we calculate these corrections both for o-Ps and p-Ps.

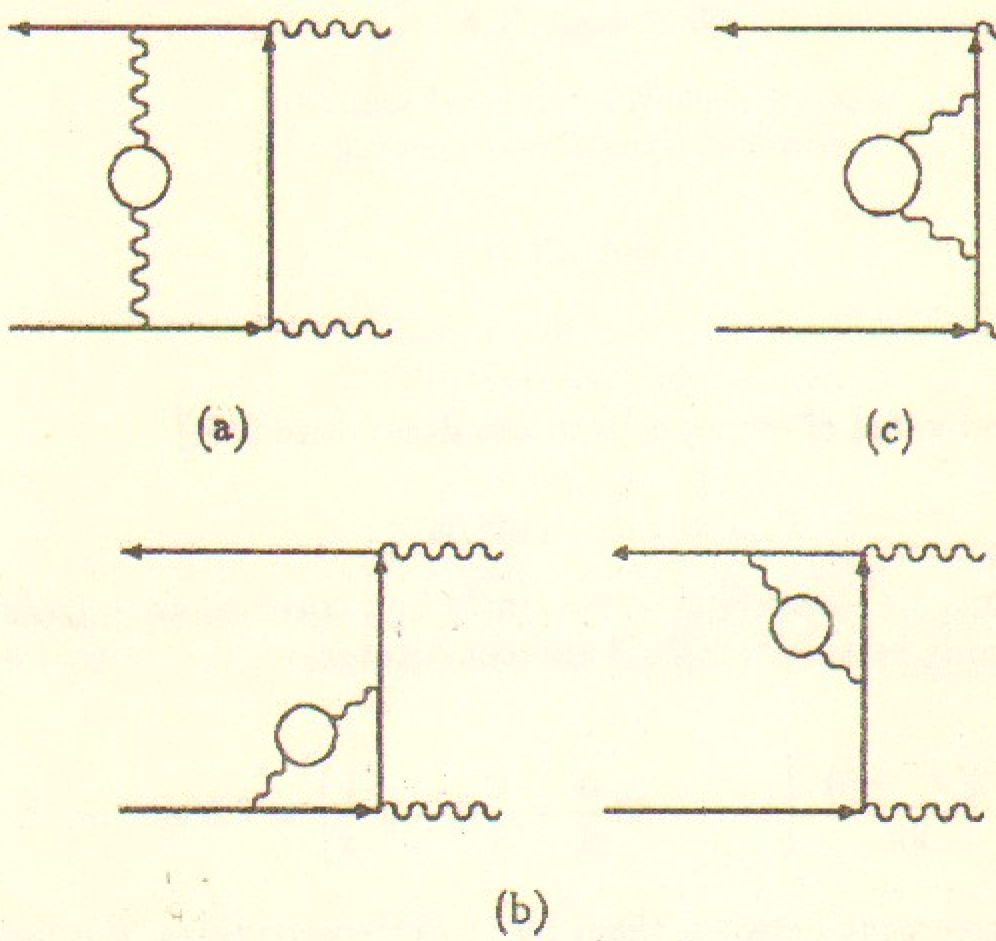

Fig. 1.

To account for the self-energy part in photon propagators, we substitute

$$\frac{g^{\mu\nu}}{k^2+i\varepsilon} \rightarrow \frac{g^{\mu\nu}}{k^2+i\varepsilon} + \int\limits_{4m^2}^{\infty} R(s)\, \frac{g^{\mu\nu}}{k^2-s+i\varepsilon} ds\,, \qquad (4)$$

(Källen—Lehmann representation); to first order in α,

$$R(s) = \frac{\alpha}{3\pi}\sqrt{\frac{s-4m^2}{s}}\,\frac{s+2m^2}{s^2}\,. \qquad (5)$$

Hence the considered contribution to positronium decay rate is of the form

$$\delta\Gamma = \int\limits_{4m^2}^{\infty} R(s)\, Q(s)\, ds\,, \qquad (6)$$

where $Q(s)$ is the one-loop correction to the decay rate calculated with the massive virtual photon ($m_\gamma = \sqrt{s}$).

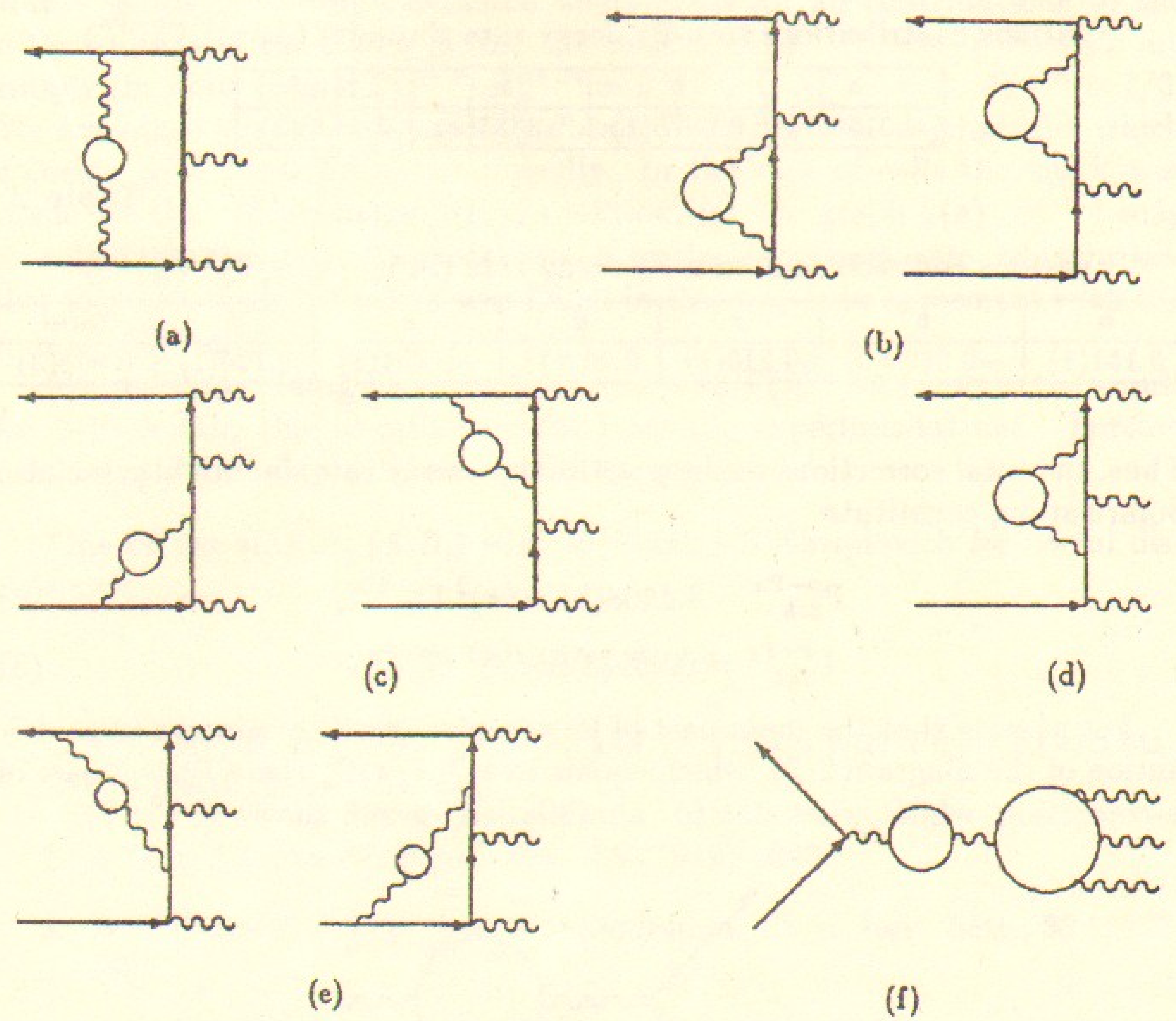

Fig. 2.

The calculation of corrections $\sim \alpha^2$ that are induced by vacuum polarization, was performed in the same manner as calculation of corrections $\sim \alpha$, which is described in the previous papers [3,5,8]. Our calculation differs from the one-loop one by the substitution (4), in which we must deal only with second term in the right-hand side, that introduced a one more integration compared to one-loop case. Appearing integrals were found numerically using convenient substitution $s = 4m^2/(1-x^2)$. To sufficient accuracy the initial particles may be considered being at rest. We use covariant summation over polarizations of the final photons. The results of computing of various diagrams are presented in Tables 1,2.

Table 1

Various contributions to p-Ps decay rate (in units $(\alpha/\pi)^2\,\Gamma_0^{p-Ps}$)

a	b	c	*total*
0.0158(1)	0.0975(1)	0.3335(1)	0.4468(3)

Table 2

Various contributions to o-Ps decay rate (in units $(\alpha/\pi)^2\,\Gamma_0^{o-Ps}$)

a	b	c	d	e	f	*total*
0.151(1)	−0.0895(3)	0.210(1)	0.062(1)	−0.094(1)	0.720(1)	0.960(3)

Thus, the total corrections to the positronium decay rate, induced by vacuum polarization, constitute

$$\Gamma_{3th}^{p-Ps} = 0.4468(3)(\alpha/\pi)^2\,\Gamma_0^{p-Ps}\,, \tag{7}$$

$$\Gamma_{3th}^{o-Ps} = 0.960(3)(\alpha/\pi)^2\,\Gamma_0^{o-Ps}\,. \tag{8}$$

Let us note that the main part of he complete result originates as contribution of the diagram $2(f)$, which equals to $-\frac{8}{9}(\alpha/\pi)\Gamma_a^1$ (here Γ_a^1 is a part of α-correction, which arises due to "annihilation" graph shown in Fig.3).

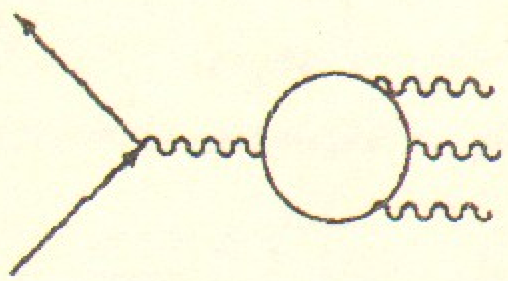

Fig. 3.

In order to check the results of the numerical calculation we also computed differential decay rate of o-Ps using three-dimensionally transverse summation over the final photons polarizations. The obtained result coincides with the differential width which was calculated employing the covariant photon polarization sums (though contributions of individual graphs do depend on manner of the summation, with the exception of the contribution of $2(f)$, which is gauge invariant by itself).

Another method to check our results is as follows. Let $T_i(s)$ to be the contribution to the $Q(s)$ due to the diagram (i) ($i = a, ..., c$ for p-Ps and $i = a, ..., f$ for o-Ps). Their asymptotics at $s \gg 1$ are $T_i(s) = A_i/s$. The values of the A_i's were calculated analytically for all contributions to the p-Ps width and for $i = b, c, d$ in the case of o-Ps. For the p-Ps A_i's are simply (in units $(\alpha/\pi)^2\,\Gamma_0^{p-Ps}$) $A_a^{p-Ps} = \pi^2/2 + 1$, $A_b^{p-Ps} = 2$, $A_c^{p-Ps} = 1/3$. We examined that the numerically obtaned T_i's have the asymptotics which coincide with those found analytically. In the case of o-Ps the check was made for the differential width. In adition, for the graph $2(a)$ A_a^{o-Ps} may be related to the part of correction $\sim \alpha$ to the o-Ps decay rate, which arises due to graph 3; A_a^{o-Ps} which was found in this way is in agreement with the numerical results.

We see that adding together Γ_{1th}, Γ_{2th} and Γ_{3th}^{o-Ps} we come to the result for o-Ps width, that is still very far from the experimental one. Further calculations probably will remove the discrepancy.

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REFERENCES

1. *J.S.Nico, D.W.Gidley, A.Rich, and P.W.Zitzewitz.* Phys. Rev. Lett., 65 (1990) 1344.
2. *A.Ore, J.L.Powell.* Phys. Rev., 75 (1949) 1696.
3. *W.E.Caswell, G.P.Lepage, J.R.Sapirstein.* Phys. Rev. Lett., 38 (1977) 488.
4. *W.E.Caswell, G.P.Lepage.* Phys. Rev., A 20 (1979) 36.
5. *G.S.Adkins.* Ann. Phys. (N.Y.) 146 (1983) 78.
6. *M.A.Stroscio, J.M.Holt.* Phys. Rev. A 10 (1974) 749.
7. *I.B.Khriplovich, A.S.Yelkhovsky.* Phys. Lett. B 246 (1990) 520.
8. *A.P.Burichenko.* Yad. Fiz. (Sov. J. Nucl. Phys.) 56 (1993) 123; BINP, Preprint 92-68.
9. *I.B.Khriplovich, A.I.Milstein, A.S.Yelkhovsky.* BINP, Preprint 92-57 (to be published in Physica Scripta).