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MIXING PHASES OF UNSTABLE TWO-LEVEL SYSTEMS

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НОВОСИБИРСК

system, its propagation and the subsequent decay. If, in general, the system contains in the given energy region N resonance levels coupled to M channel states the matrix $\mathcal{A}$ is a rectangular N x M-matrix composed of the transition amplitudes $\mathcal{A}_m^c$ between internal states $\mid m>(m=1,2,\ldots,N)$ and channel states $\mid c>(c=1,2,\ldots,M)$. These amplitudes can be considered within limited energy intervals distant from thresholds as energy-independent quantities. They are real provided that T-invariance holds.

The evolution of the intermediate unstable system is described according to eq. (1) by the non-hermitian matrix

$$\mathcal{H}=H-\frac{i}{2}W\,, \qquad (2)$$

which can be considered as an effective Hamiltonian [6-8]. This Hamiltonian acts only within the intrinsic N-dimensional space but acquires, due to elimination of the degrees of freedom of motion in the energy continuum, an antihermitian part. The hermitian part H is the internal Hamiltionian with a discrete spectrum whereas the antihermitian part W originates from on-shell self-energy contributions corresponding to open decay channels. By this, one gains the ability of a discretized treatment of the intrinsic dynamics of systems embedded in continuum.

Both the matrices H and W are real and symmetric in T-invariant theory. The effective Hamiltonian $\mathcal{H}$ is therefore also symmetric. Due to unitarity of the scattering matrix, the antihermitian part W is expressed [6-8] in the specific factorized form

$$W=\mathcal{A}\mathcal{A}^T \qquad (3)$$

in terms of the same transition amplitudes $\mathcal{A}_m^c$ which appear in the reaction matrix (1). All parameters of the resonance scattering amplitudes $T^{cc'}$ are hence presented in the effective Hamiltonian (2)-(3). This effective Hamiltonian plays an important role in the theory of resonance reactions [8].

The eigenenergies and eigenstates of the effective Hamiltonian are of special interest. They are found from matrix equation

$$\mathcal{H}\Psi=\Psi\mathcal{E}\,, \qquad (4)$$

where $\mathcal{E}$ is the diagonal matrix of complex energies $\mathcal{E}_m=E_m-\frac{i}{2}\Gamma_m$ with E_m and Γ_m being the energy and width of m-th resonance state. Such states form the columns of the N x N matrix Ψ. Inasmuch as the matrix of effercive Hamiltonian $\mathcal{H}$ is symmetric, the matrix of eigenstates Ψ can be chosen to be complex orthogonal [8]

$$\Psi^T\Psi=\Psi\Psi^T=1\,. \qquad (5)$$

Note that the normalization of eigenstates $\sum_m\Psi_m^2=1$ following from (5) differs from the condition $\sum_m|\Psi_m|^2=1$ used in the elementary particle physics [3] and corresponds to that usually adopted in the theory of nuclear reactions [9]. We found this choice to be more convenient for our purposes.

Contrary to (5), the matrix

$$U=\Psi^+\Psi \qquad (6)$$

is not the unit one. Decaying states are not, in general, orthogonal. It is easy to prove [8] that

$$U\mathcal{E}-\mathcal{E}^+U=-i\Psi^+W\Psi\equiv-i\tilde{\mathcal{A}}^*\tilde{\mathcal{A}}^T\,. \qquad (7)$$

This is a matrix version of the well-known Bell-Steinberger relation [3]. Unlike $\mathcal{A}$, the tilted matrix

$$\tilde{\mathcal{A}}=\Psi^T\mathcal{A} \qquad (8)$$

is complex due to complexity of the eigenvectors of the effective Hamiltonian. The complex matrix elements $\tilde{\mathcal{A}}_m^c$ play the role of the decay amplitudes of unstable states. The on- and off-diagonal elements of eq. (7) are

$$\Gamma_m=\frac{1}{U_{mm}}\sum_c\left|\tilde{\mathcal{A}}_m^c\right|^2\equiv\sum_c\Gamma_m^c\,, \qquad (9)$$

$$\sum_c\tilde{\mathcal{A}}_m^{c*}\tilde{\mathcal{A}}_n^c=i\,(\mathcal{E}_n-\mathcal{E}_m^*)\,U_{mn}\,. \qquad (10)$$

The quantities Γ_m^c are the partial widths by definition.

The matrix of the eigenstates Ψ is used to represent the reaction amplitudes eq. (1) in the explicit resonance form

$$T^{cc'}(E)=\sum_m\frac{\tilde{\mathcal{A}}_m^c\tilde{\mathcal{A}}_m^{c'}}{E-\mathcal{E}_m}\,. \qquad (11)$$

Only one term in the sum (11) dominates in the vicinity of a given reaction energy E provided that resonances are well separated i.e. the widths of resonances are much less than spacings between them. It can be easily seen that in this case all eigenstates of the effenctive Hamiltonian are real, the matrix (6) coincides with the unit matrix and the amplitudes $\tilde{\mathcal{A}}_m^c=\sqrt{\Gamma_m^c}$ are real. The energy dependence of cross-sections has therefore the standard

Breit-Wigner form defined by the energy of the resonance and its partial widths.

In the opposite case of overlapping resonances, the amplitudes

$$\tilde{\mathcal{A}}_m^c \equiv \sqrt{\Gamma_m^c}\sqrt{U_{mm}}e^{i\alpha_m^c} \tag{12}$$

are complex and the reaction energy spectrum is formed by a non-trivial interference of different terms in the sum (11) corresponding to these resonances. This spectrum can not be analysed in term of energies and partial widths only. The mixing phases α_m^c play in this analysis an important role.

In what follows we will consider the decay properties in the special case of a system with only two overlapping resonances. Various aspects of this problem have attracted attention before this work [7,10,12,13,14]. Recently, these properties were discussed in the frame of the effective Hamiltonian approach in a number of papers [15-17] by one of the authors in the special case when only one decay channel is open. In the present paper, we extend this consideration to an arbitrary number of open channels. We will show that, for any number of channels, the mixing phases α_m^c can be eliminated from the decay energy spectrum of an unstable two-level system and this spectrum is expressed only in terms of energies and partial widths of resonances and one additional universal parameter which satisfies a sum rule following from the Bell-Steinberger relation (10).

Two-level system

1. The orthogonal matrix of eigenvectors of a 2x2 symmetric matrix coincides with the well-known matrix of rotation in a plane provided that this symmetric matrix is real. The matrix Ψ depends then on the only real parameter, the angle of rotation in the plane. To diagonalize a complex symmetric matrix one needs a complex orthogonal matrix which still can be presented in the same form of plane rotation

$$\Psi = \begin{pmatrix} \cos\chi & \sin\chi \\ -\sin\chi & \cos\chi \end{pmatrix} \tag{13}$$

but with complex angle $\chi = \chi_2 + i\chi_2$. The imaginary part χ_2 plays an important role describing the special mixtures of the originally bounded states which forms the unstable (resonance) states under consideration. Another parametrization which is frequently used in particle physics [3,7,10-13] looks

like

$$\Psi = \frac{1}{\sqrt{1+|\epsilon|^2}}\begin{pmatrix} 1 & -\epsilon \\ \epsilon & 1 \end{pmatrix}. \tag{14}$$

After corresponding renormalization of eigenvectors the two parameters can be connected by the simple formula $\epsilon = \tan\chi$.

The complex decay amplitudes $\tilde{\mathcal{A}}_m^c$ are therefore

$$\begin{aligned} \tilde{\mathcal{A}}_1^c &= \cos\chi\mathcal{A}_1^c - \sin\chi\mathcal{A}_2^c\,, \\ \tilde{\mathcal{A}}_2^c &= \sin\chi\mathcal{A}_1^c + \cos\chi\mathcal{A}_2^c\,. \end{aligned} \tag{15}$$

Since the quantities $\mathcal{A}_m^c$ are real, all phases α_m^c are governed by the parameter χ_2 only. It leads to two conditions

$$\begin{aligned} \sinh\chi_2(Re\tilde{\mathcal{A}}_1^c) - \cosh\chi_2(Im\tilde{\mathcal{A}}_2^c) &= 0\,, \\ \cosh\chi_2(Im\tilde{\mathcal{A}}_1^c) + \sinh\chi_2(Re\tilde{\mathcal{A}}_2^c) &= 0\,. \end{aligned} \tag{16}$$

$$\begin{aligned} \frac{\mathrm{Im}\tilde{\mathcal{A}}_1^c}{\mathrm{Re}\tilde{\mathcal{A}}_1^c}\frac{\mathrm{Im}\tilde{\mathcal{A}}_2^c}{\mathrm{Re}\tilde{\mathcal{A}}_2^c} &= -\tanh^2\chi_2\,, \\ \frac{\mathrm{Re}\tilde{\mathcal{A}}_1^c}{\mathrm{Re}\tilde{\mathcal{A}}_2^c}\frac{\mathrm{Im}\tilde{\mathcal{A}}_1^c}{\mathrm{Im}\tilde{\mathcal{A}}_2^c} &= -1 \end{aligned} \tag{17}$$

are therefore channel-independent. Using the definition (12) and the fact that in the two-resonance case $U_{11} = U_{22}$ (see eq.(6) and eq.(21) below) one easily obtains the equations

$$\begin{aligned} \tan\alpha_1^c\tan\alpha_2^c &= -\tanh^2\chi_2, \\ (\Gamma_1^c - \tanh^2\chi_2\Gamma_2^c)\tan\alpha_1^c &+ (\Gamma_2^c - \tanh^2\chi_2\Gamma_1^c)\tan\alpha_2^c = 0\,, \end{aligned} \tag{18}$$

which give

$$\begin{aligned} \tan^2\alpha_1^c &= \frac{\Gamma_2^c - \tanh^2\chi_2\Gamma_1^c}{\Gamma_1^c - \tanh^2\chi_2\Gamma_2^c}\tanh^2\chi_2\,, \\ \tan^2\alpha_2^c &= \frac{\Gamma_1^c - \tanh^2\chi_2\Gamma_2^c}{\Gamma_2^c - \tanh^2\chi_2\Gamma_1^c}\tanh^2\chi_2\,. \end{aligned} \tag{19}$$

One of the combinations of the partial widths in the fractions in r.h.s. of eqs. (19) is certainly positive. For eqs. (19) to be consistent, the other combination must be also positive. It should be provided by the value of χ_2 which is not a free parameter but is determened by the sum rule given below (see eq. (24)).

The absolute signs of $\tan\alpha_m^c$ are left uncertain by eqs. (19) though we can see from the first relation in eqs. (18) that they are opposite for the two resonances. The cross-section and the decay energy spectrum are expressed in the terms of sine and cosine of the phase differences $\alpha_1^c - \alpha_2^c$ which can be restricted to the interval $[0,\pi]$. The sine is positive in this interval whereas the cosine can be positive or negative depending on whether the phase difference is less than $\frac{\pi}{2}$ or exceeds this value. Only the absolute value of the $\cos(\alpha_1^c - \alpha_2^c)$ is fixed however by eq.(18). With all this taken into account, we arrive from the eqs. (18), (19) at

$$\begin{aligned} \sin(\alpha_1^c - \alpha_2^c) &= -\frac{1}{2}\frac{\Gamma_1^c + \Gamma_2^c}{\sqrt{\Gamma_1^c\Gamma_2^c}}\tanh 2\chi_2\,, \\ \cos(\alpha_1^c - \alpha_2^c) &= \pm\frac{\sqrt{(\Gamma_1^c - \tanh^2\chi_2\Gamma_2^c)(\Gamma_2^c - \tanh^2\chi_2\Gamma_1^c)}}{(1+\tanh^2\chi_2)\sqrt{\Gamma_1^c\Gamma_2^c}}\,. \end{aligned} \tag{20}$$

Two solutions symmetric with respect to the value $\frac{\pi}{2}$ are given by (20) for each phase difference $\alpha_1^c - \alpha_2^c$. One can not choose one of them on this stage.

Let us now make use the Bell-Steinberger relation (10). With the parametrization (13), the non-orthogonality matrix (6) is

$$U = \begin{pmatrix} \cosh 2\chi_2 & i\sinh 2\chi_2 \\ -i\sinh 2\chi_2 & \cosh 2\chi_2 \end{pmatrix}. \tag{21}$$

The relation (10) is therefore equivalent to the two conditions

$$\sum_c \sqrt{\Gamma_1^c\Gamma_2^c}\sin(\alpha_1^c - \alpha_2^c) = -\frac{1}{2}(\Gamma_1 + \Gamma_2)\tanh 2\chi_2\,, \tag{22}$$

$$\sum_c \sqrt{\Gamma_1^c\Gamma_2^c}\cos(\alpha_1^c - \alpha_2^c) = (E_1 - E_2)\tanh 2\chi_2\,. \tag{23}$$

The first of them is identically satisfied by eq. (20) whereas the condition (23) gives the non-trivial sum rule which can be reduced after some algebra to

$$\sum_c S_c\sqrt{\Gamma_1^c\Gamma_2^c - \frac{1}{4}\sinh^2 2\chi_2\,(\Gamma_1^c - \Gamma_2^c)^2} = (E_1 - E_2)\sinh 2\chi_2\,. \tag{24}$$

Here, the signs S_c in front of the square roots are still uncertain in accordance with the second equation (20). However, since the value of physical parameter χ_2 is unique, all signs in (24) should be as well determined in selfconsistent way by the same sum rule. Some additional information can be also used in a concrete situation (see, for example, discussion of the $\rho - \omega$ mixing below). The numeration of the resonances is chosen in such a way that $(E_2 - E_1) > 0$. The equation (24) defines the parameter χ_2 in the terms of the spacing between the resonances and their partial widths.

2. The effective Hamiltonian $\mathcal{H}$ of two-resonance system being a 2x2 complex symmetric matrix is determined by 6 real combinations of the S-matrix parameters. In general, it can be represented in the form

$$\mathcal{H} = \begin{pmatrix} \epsilon_1 - \frac{i}{2}\gamma_1 & \frac{1}{2}\delta\epsilon \\ \frac{1}{2}\delta\epsilon & \epsilon_2 - \frac{i}{2}\gamma_2 \end{pmatrix} - \frac{i}{2}\begin{pmatrix} 0 & \sqrt{\gamma_1\gamma_2}\cos\beta \\ \sqrt{\gamma_1\gamma_2}\cos\beta & 0 \end{pmatrix}, \tag{25}$$

where two initial "unperturbed" resonances with complex energies $\epsilon_m - \frac{i}{2}\gamma_m$ are mixed by the internal interaction $\frac{1}{2}\delta\epsilon$ as well as by the external interaction with amplitudes $\mathcal{A}_m^c$ via the continuum;

$$\gamma_m = \sum_c (\mathcal{A}_m^c)^2 \quad ; \quad \cos\beta = \frac{\sum_c \mathcal{A}_1^c\mathcal{A}_2^c}{\sqrt{\gamma_1\gamma_2}}. \tag{26}$$

The six elements of the matrix (25) are connected by the formula

$$\mathcal{H} = \Psi\mathcal{E}\Psi^T \tag{27}$$

to the complex energies of the two resonances $\mathcal{E}_1$ and $\mathcal{E}_2$ (4 real parameters) and the complex angle χ introduced in the previous point (2 real parameters). Five of these parameters, the energies and widths as well as the imaginary part χ_2, are invariant under arbitrary real orthogonal transformation of the intrinsic basis. As we have shown above, the parameter χ_2 is defined by the energy spliting and partial widths of resonances. Contrary to this, the sixth parameter, the real angle χ_1, depends on the choice of the basis of intrinsic motion. This choice is dictated by physical reasons. We stress that the real decay amplitudes $\mathcal{A}_m^c$ are also basis-dependent; decays of actual resonances are described by the complex amplitudes $\tilde{\mathcal{A}}_m^c$.

Two possible choices of the intrinsic basis are of special interest. One can diagonalize by some real orthogonal transformation either the hermitian or the anti-hermitian part of $\mathcal{H}$. In the first case $\delta\epsilon = 0$ and two unperturbed resonances are mixed by a purely imaginary interaction via the continuum ("external mixing"). The condition $\delta\epsilon = 0$ fixes angle χ_1 to satisfy the condition

$$\tan 2\chi_1 = -\frac{1}{2}\frac{\Gamma_1 - \Gamma_2}{E_1 - E_2}\tanh 2\chi_2 . \qquad (28)$$

The five remaining matrix elements in (25) can then be easily expressed in terms of the complex energies $\mathcal{E}_1$ and $\mathcal{E}_2$ and of the parameter χ_2 of "physical" resonance states. One gets

$$\begin{aligned}(\epsilon_1 - \epsilon_2)^2 &= \left[(E_1 - E_2)^2 + \frac{1}{4}(\Gamma_1 - \Gamma_2)^2 \tanh^2 2\chi_2\right]\cosh^2 2\chi_2,\\ (\gamma_1 - \gamma_2)^2 &= \frac{(E_1 - E_2)^2 (\Gamma_1 - \Gamma_2)^2}{\left[(E_1 - E_2)^2 + \frac{1}{4}(\Gamma_1 - \Gamma_2)^2 \tanh^2 2\chi_2\right]\cosh^2 2\chi_2},\end{aligned} \qquad (29)$$

$$\gamma_1\gamma_2\cos^2\beta = \frac{\left[(E_1 - E_2)^2 + \frac{1}{4}(\Gamma_1 - \Gamma_2)^2\right]^2}{\left[(E_1 - E_2)^2 + \frac{1}{4}(\Gamma_1 - \Gamma_2)^2 \tanh^2 2\chi_2\right]}\sinh^2 2\chi_2, \qquad (30)$$

whereas the two equalities

$$\epsilon_1 + \epsilon_2 = E_1 + E_2, \qquad \gamma_1 + \gamma_2 = \Gamma_1 + \Gamma_2 \qquad (31)$$

are provided by the invariance of matrix trace.

In the second case, $\cos\beta = 0$ and the initial resonances are mixed by a real internal interaction ("internal mixing"). The angle χ_1 is now given by

$$\tan 2\chi_1 = 2\frac{E_1 - E_2}{\Gamma_1 - \Gamma_2}\tanh 2\chi_2 \qquad (32)$$

and

$$\begin{aligned}(\epsilon_1 - \epsilon_2)^2 &= \frac{1}{4}\frac{(E_1 - E_2)^2 (\Gamma_1 - \Gamma_2)^2}{\left[(E_1 - E_2)^2 \tanh^2 2\chi_2 + \frac{1}{4}(\Gamma_1 - \Gamma_2)^2\right]\cosh^2 2\chi_2},\\ (\gamma_1 - \gamma_2)^2 &= 4\left[(E_1 - E_2)^2 \tanh^2 2\chi_2 + \frac{1}{4}(\Gamma_1 - \Gamma_2)^2\right]\cosh^2 2\chi_2,\end{aligned} \qquad (33)$$

$$(\delta\epsilon)^2 = \frac{\left[(E_1 - E_2)^2 + \frac{1}{4}(\Gamma_1 - \Gamma_2)^2\right]^2}{\left[(E_1 - E_2)^2 \tanh^2 2\chi_2 + \frac{1}{4}(\Gamma_1 - \Gamma_2)^2\right]}\sinh^2 2\chi_2 . \qquad (34)$$

Note that, as it can be easily seen from eqs. (29) and (33), the attraction-repulsion theorems established in [16] for the single-channel case remain valid also for arbitrary number of channels (see also [15]).

3. Additional simplifications emerge if only one decay channel is open since the partial widths of the resonances coincide in this case with the total widths. One can therefore find the parameter χ_2 explicitly. To do this, one can take into account that one of the initial widths γ vanishes in the internal mixing basis whereas another is equal to sum $\Gamma_1 + \Gamma_2$ [8]. It gives

$$\tanh^2 2\chi_2 = \frac{\Gamma_1\Gamma_2}{(E_1 - E_2)^2 + \frac{1}{4}(\Gamma_1 + \Gamma_2)^2} . \qquad (35)$$

The formulae (29), (30) and (33), (34) are reduced after substitution of eq. (35) to the corresponding expressions of the ref. [16]. In particular,

$$(\delta\epsilon)^2 = 4\frac{\Gamma_1\Gamma_2}{(\Gamma_1 + \Gamma_2)^2}\left[(E_1 - E_2)^2 + \frac{1}{4}(\Gamma_1 + \Gamma_2)^2\right] . \qquad (36)$$

We will use this equality in the next section. At last the mixing phase can be easily found to be [7]

$$\tan(\alpha_1 - \alpha_2) = \frac{1}{2}\frac{\Gamma_1 + \Gamma_2}{E_2 - E_1} . \qquad (37)$$

Applications

The resonances mixed due to violation of an approximate symmetry represent nice special patterns of overlapping resonances. The isospin symmetry or CP-symmetry broken in the decays of the neutral K-mesons are typical examples. The initial symmetry prompts a natural choice of the intrinsic basis in these cases.

a) 2^+ doublett in ^{8}Be

The well-known doublett of 2^+ states of ^{8}Be with energies 16,7 and 17,0MeV decaying into the two-α-particle channel with zero isospin are formed by two states with isospins 1 and 0. These states are considered to be composed from the two charge conjugate states

$$|^7Li+p\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad |^7Be+n\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} . \qquad (38)$$

From the charge symmetry one expects that the latter states have nearly equal energies and decay amplitudes. In the limit of the exact charge symmetry Hamiltonian matrix (25) has in the basis of constituents (38) the form

$$\mathcal{H} = \begin{pmatrix} \epsilon_0 & v \\ v & \epsilon_0 \end{pmatrix} - \frac{i}{4} \begin{pmatrix} \gamma_0 & \gamma_0 \\ \gamma_0 & \gamma_0 \end{pmatrix} \qquad (39)$$

clearly manifesting this symmetry.

The matrix (39) can be diagonalized by the real orthogonal transformation given by the matrix

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \qquad (40)$$

of eigenvectors

$$\begin{aligned} |1\rangle &= \frac{1}{\sqrt{2}}\left(|^7Li+p\rangle - |^7Be+n\rangle\right) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \\ |0\rangle &= \frac{1}{\sqrt{2}}\left(|^7Li+p\rangle + |^7Be+n\rangle\right) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{aligned} \qquad (41)$$

with isospins 1 and 0 correspondingly. The diagonal form

$$\mathcal{H} = \begin{pmatrix} \epsilon - v & 0 \\ 0 & \epsilon + v - \frac{i}{2}\gamma_0 \end{pmatrix} \qquad (42)$$

directly reveals the isospin conservation due to the underlying charge symmetry: the state with isospin 1 is stable.

Let us now suppose that the charge symmetry is broken only in the energies of the constituents (38) whereas their decay amplitudes remain equal. Then the effective Hamiltonian is perturbed by the energy-shift matrix

$$\delta H = \begin{pmatrix} \frac{1}{2}\delta\epsilon & 0 \\ 0 & -\frac{1}{2}\delta\epsilon \end{pmatrix} \longrightarrow \begin{pmatrix} 0 & \frac{1}{2}\delta\epsilon \\ \frac{1}{2}\delta\epsilon & 0 \end{pmatrix} \qquad (43)$$

and gains therefore in the basis (41) the internal-mixing form given by the sum of eq. (42) and r.h.s. of eq. (43). Using the formula (36) of the previous

section and the experimental data by Bonn group [18] we can find the energy difference of the constituent states (38)

$$|\delta\epsilon| = 296.6\,\text{keV} . \qquad (44)$$

b) The system of ρ and ω mesons

The system of the two neutral vector mesons ρ^0 and ω^0 is another example of resonance mixing caused by violation of the isospin conservation [19]. These mesons are composed of the two two-quark states, $|u\bar{u}\rangle$ and $|d\bar{d}\rangle$, which can decay into a number of channels, the 2π- and 3π- channels being the most important ones. Two orthogonal states with opposite G-parities and isospins 1 and 0 decaying only into 2π- and 3π channels are formed provided that the light quarks have equal masses and one neglects the electro-weak interaction. In the basis of these states the matrix of real decay amplitudes looks like

$$\mathcal{A} = \begin{pmatrix} \sqrt{\gamma_1} & 0 \\ 0 & \sqrt{\gamma_0} \end{pmatrix} , \qquad (45)$$

where we have used the value of isospin to label the states and channels.

The mass shift $\delta m{=}\delta\epsilon$ of the constituent states due to different masses of u and d quarks breaks the isotopic symmetry and forms the two mixed physical states $|\rho\rangle$ and $|\omega\rangle$. Suggesting that this shift is the only cause of symmetry breaking one finds from eqs. (45) and (15) the complex decay amplitudes

$$\begin{aligned} \tilde{\mathcal{A}}_\rho^{2\pi} &= \cos\chi\sqrt{\gamma_1} & \tilde{\mathcal{A}}_\rho^{3\pi} &= -\sin\chi\sqrt{\gamma_0} \\ \tilde{\mathcal{A}}_\omega^{2\pi} &= \sin\chi\sqrt{\gamma_1} & \tilde{\mathcal{A}}_\omega^{3\pi} &= \cos\chi\sqrt{\gamma_0} . \end{aligned} \qquad (46)$$

It leads immediately to

$$\tan\left(\alpha_\rho^{3\pi} - \alpha_\omega^{3\pi}\right) = -\tan\left(\alpha_\rho^{2\pi} - \alpha_\omega^{2\pi}\right) \qquad (47)$$

and

$$\Gamma_\rho^{3\pi} = \frac{\Gamma_\omega^{2\pi}}{\Gamma_\rho^{2\pi}}\Gamma_\omega^{3\pi} \approx 0.63 \cdot 10^{-4}\Gamma_\rho . \qquad (48)$$

These relations have good accuracy in the frame of the adopted mechanism of symmetry breaking since they can be violated only by electromagnetic corrections, which can create small off-diagonal elements in the amplitude matrix (45). The width $\Gamma_\rho^{3\pi}$ is not known experimentally. In view of our discussion and the unambigous prediction (47) it would be interesting to measure this width. This can be done presumably at the planed DaΦne accelerator.

The established relations (47), (48) allow one to solve exactly the equation (24). In particular , the second connection determines the relative sign of the two terms in this sum rule. Up to the terms of higher order on the ratio $\frac{\Gamma_\omega^{2\pi}}{\Gamma_\rho}$ one obtains

$$\sinh^2 2\chi_2 = \frac{(\Gamma_\rho - \Gamma_\omega)^2}{(m_\omega - m_\rho)^2 + \frac{1}{4}(\Gamma_\rho - \Gamma_\omega)^2} \frac{\Gamma_\omega^{2\pi}}{\Gamma_\rho} . \qquad (49)$$

It leads to the expression for the mixing phases

$$\tan\left(\alpha_\rho^{2\pi} - \alpha_\omega^{2\pi}\right) = -\tan\left(\alpha_\rho^{3\pi} - \alpha_\omega^{3\pi}\right) = \frac{1}{2}\frac{\Gamma_\rho - \Gamma_\omega}{m_\omega - m_\rho} \approx 5.15 , \qquad (50)$$

which differs from that given in ref.[13] by sign in front of the width of ω-meson. We stress in this connection that the accuracy of the eq. (50) is $\frac{\Gamma_\omega^{2\pi}}{\Gamma_\rho}$ (or, more exactly, $\frac{\Gamma_\omega^{\pi^0\gamma}}{\Gamma_\rho}$ with electomagnetic decays taken into account) rather than $\frac{\Gamma_\omega}{\Gamma_\rho}$.

Since the matrix (45) of the amplitudes is diagonal the effective Hamiltonian of the $\rho - \omega$ system has in the adopted approximation the internal mixing form again. The formulae (34) and (49) give for the mixing mass shift

$$|\delta m| = 2\left[(m_\omega - m_\rho)^2 + \frac{1}{4}(\Gamma_\rho - \Gamma_\omega)^2\right]^{\frac{1}{2}} \left(\frac{\Gamma_\omega^{2\pi}}{\Gamma_\rho}\right)^{\frac{1}{2}} \approx 5.07\,\text{MeV} . \qquad (51)$$

This value is only 7% less than the estimate of ref.[17] obtained by a somewhat tricky one-channel approximation.

We have considered an unstable two-level system decaying into a number of open channels. It is shown that the mixing phases of the two overlapping resonances can be obtained from their partial width and one additional mixing parameter. Applications are made to the doublett of resonances in ^{8}Be and to the $\rho-\omega$-system. In particular one obtains the 3π decay width of the ρ meson to be $\Gamma_\rho^{3\pi} = 0.6310^{-4}\Gamma_\rho$.

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