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ON THE PARAXIAL EQUILIBRIUM OF THE FINITE β PLASMA IN OPEN MAGNETIC CONFIGURATIONS

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On the paraxial equilibrium of the finite β plasma in open magnetic configurations

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ABSTRACT

Paraxial equilibrium of the finite β plasma in the magnetic mirror is considered. The upper β limit is established, upwards of which a current layer is generated in plasma, and, as a consequence, a small longitudinal scale-length occurs in the problem. The latter leads to the violation of the paraxial equilibrium. Results are applicable to various plasma configurations with the mirror confinement.

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Consider a paraxial (or "long-thin") magnetic mirror with the vacuum magnetic field on the axis $B_v(z)$, increasing from the middle plane ($z = 0$) to the plugs. Let $\mathcal{R}$ be the mirror ratio of the vacuum field. In the paraxial approximation, when the characteristic radius a of the plasma is small as compared with the scale-length of the magnetic field variation along the axis, the transverse equilibrium equation claims (see, e.g., the survey [1]):

$$8\pi p_\perp + B^2 = B_v^2(z)\,, \tag{1}$$

where B is the field strength, $p_\perp$ is the perpendicular plasma pressure, which can be expressed as

$$p_\perp = \sum 2\sqrt{2}\pi M^{-3/2} B^2 \int_0^\infty d\varepsilon \int_{\varepsilon/B_*}^{\varepsilon/B} d\mu\, f\, \mu(\varepsilon - \mu B)^{-1/2}\,. \tag{2}$$

Here f denotes the guiding center distribution function, ε and μ are the energy and the magnetic moment of the particle, B_* is the maximum field strength in the plug, and the summation over the particle species is carried out.

In the collisionless limit, f is a function of only ε and μ on the fixed field line (see [2]). Since the particles with various energies contribute to the pressure integral in an additive way, one can analyze the equilibrium for the monoenergetic distribution

$$f = F(\mu)\,\delta(\varepsilon - \varepsilon_0)\,, \tag{3}$$

and then perform the summation over all groups of particles. In this paper we select the function $F(\mu)$ in the form:

$$F(\mu) = A\left(\frac{\mu - \mu_*}{\mu_*}\right), \tag{4}$$

where $\mu_* = \varepsilon/B_*$, and A is a factor, proportional to the number of particles on the given field line. The model profile (4) corresponds to the plasma distribution, in which particles with large pitch angles dominate. Besides, the function (4) vanishes on the surface of the loss cone, which is determined by the equation $\mu = \mu_*$.

Let us introduce β as the ratio between the perpendicular plasma pressure and the pressure of the vacuum field in the midplane:

$$\beta = \left.\frac{8\pi p_\perp}{B_v^2}\right|_{z=0}. \tag{5}$$

The computation of $p_\perp$, using (2)-(4), leads, accounting for (1), to the equilibrium equation

$$G(R) = R_v^2(z), \tag{6}$$

where

$$G(R) \equiv \eta\frac{(1-R)^{3/2}(R+4)}{R} + R^2, \tag{7}$$

and the following notations are accepted:

$$R \equiv \frac{B}{B_*}, \qquad R_v \equiv \frac{B_v}{B_*},$$

$$\eta \equiv \frac{\beta R_{v0}^3(1-\beta)^{1/2}}{\left[(1 - R_{v0}(1-\beta)^{1/2}\right]^{3/2}\left[4 + R_{v0}(1-\beta)^{1/2}\right]}. \tag{8}$$

The subscript "0" refers to the values in the midplane (so that R_{v0}, for instance, equals to the inverse vacuum mirror ratio, $R_{v0} = \mathcal{R}^{-1}$). Expression for the pressure $p_\perp$ cancels at the point $B = B_*$ (the first term in (7) equals to zero if $R = 1$). In accordance with (5), (6), the quantities R_0 and R_{v0} are related as

$$R_0 = R_{v0}(1-\beta)^{1/2}. \tag{9}$$

The plot of the function $G(R)$ at some fixed values R_{v0} and β is presented on Fig.1. The specific feature of the function G is that there exists a point $R_{min}(R_{v0}, \beta)$, where G has its minimum. One can show that if

$$R_0 < R_{min}, \tag{10}$$

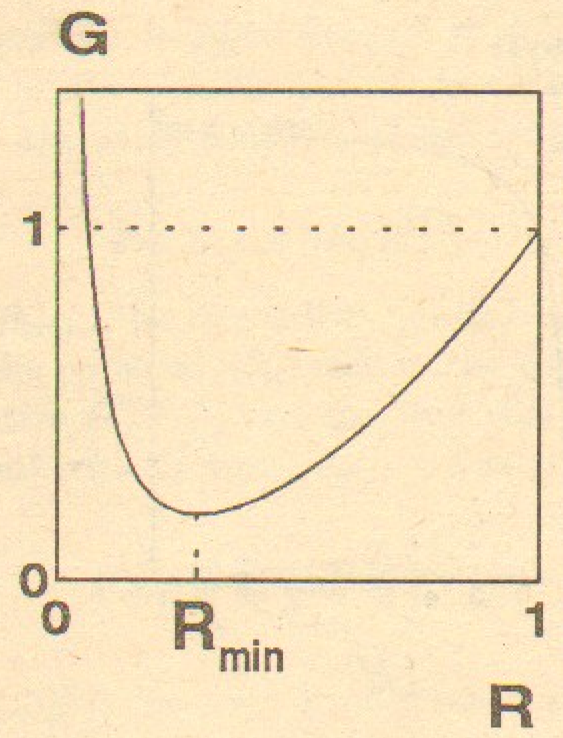

Figure 1: $G(R)$ profile.

then the equilibrium equation (6) has no solutions. Indeed, if (10) is satisfied, then the function $G(R)$ grows in the midplane direction in the interval $R_0 < R < R_{min}$. The latter results in a contradiction (see (6)) with the assumption of the decrease of the profile $R_v(z)$ in the midplane direction.

As follows from the elementary analysis, in the small β limit the value R_0 always exceeds that of R_{min}. As β grows (with R_{v0} being fixed), R_0 decreases, whereas R_{min} increases. At some value $\beta = \beta_{crit}$ the quantities R_0 and R_{min} become equal to each other. With the further growth of β, the inequality (10) comes into force, and the paraxial equilibrium becomes impossible. Thus, the critical value β_{crit} is determined by the requirement

$$\left.\frac{dG}{dR}\right|_{R=R_0} = 0, \tag{11}$$

or

$$-\frac{1}{2}\eta\frac{(1-R_0)^{1/2}}{R_0^2}(3R_0^2 + 4R_0 + 8) + 2R_0 = 0. \tag{12}$$

The numerical solution of the equation (12) is presented on Fig.2, where β_{crit} as a function of $\mathcal{R}$ being shown.

In the limiting cases

$$\mathcal{R} - 1 \ll 1,$$

$$\mathcal{R} \gg 1,$$

β_{crit}

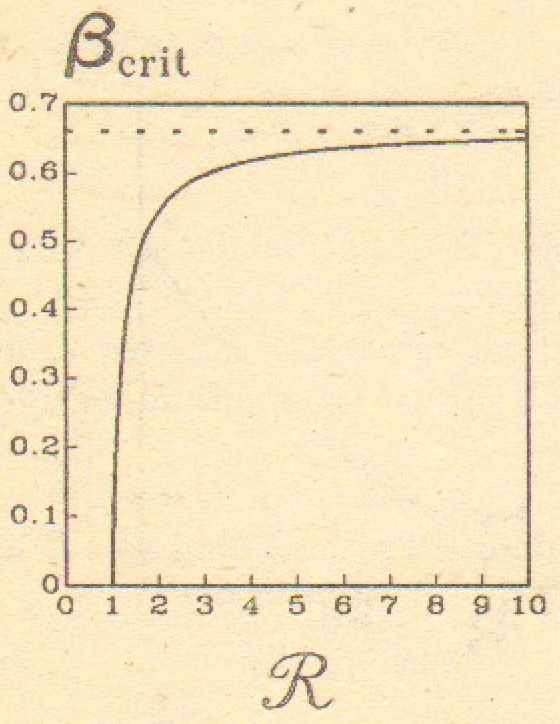

Figure 2: β_{crit} versus vacuum mirror ratio $\mathcal{R}$. The dotted line marks the maximum value $\beta_{crit} = 2/3$.

the asymptotes of the function $\beta_{crit}(\mathcal{R})$ are found to be

$$\beta_{crit} = 4(\mathcal{R} - 1)\,,$$

and

$$\beta_{crit} = \frac{2}{3}\,,$$

respectively.

Examine now the magnetic field behavior in the vicinity of the midplane, where B can be written as

$$B_0 + b\,,$$

with b being a small correction to the value B_0 in the midplane. The expansion of the vacuum field near this plane gives:

$$R_{v0}\left(1 + \frac{1}{2}\frac{z^2}{L^2}\right)\,,$$

where L is the characteristic scale-length of the vacuum field variation in the axial direction. Using (6), one obtains:

$$\frac{b}{B_0} = \frac{z^2}{L^2}\left(\mathcal{R}G'(R_0)\frac{R_0}{R_{v0}}\right)^{-1}\,, \qquad (13)$$

with the prime denoting the R-derivative, $G' \equiv dG/dR$. According to (9), $R_0/R_{v0} \gtrsim 1$ (since $\beta < \beta_{crit} \gtrsim 1$), and so the relation (13) leads to the following estimate for the scale-length l of the magnetic field B variation along the axis:

$$l \sim L\,[\mathcal{R}G'(R_0)]^{1/2}\,. \qquad (14)$$

When β approaches β_{crit}, the derivative $G'(R_0)$ tends to zero (see (11)) together with l, and the paraxial approach in the equilibrium problem fails. Evaluating the derivative $G'(R_0)$, one can find that the dependence of the scale-length l on the subcriticity value $\beta_{crit} - \beta$ is given by the relation

$$l \sim L\left(\frac{\beta_{crit} - \beta}{\beta_{crit}}\right)^{1/2}\,. \qquad (15)$$

The paraxial approximation becomes invalid when the magnetic field variation in the axial direction has a scale-length, comparable with the characteristic plasma radius a. Accounting for (15), one obtains that the paraxial equilibrium violates even for

$$\beta_{crit} - \beta \gtrsim \beta_{crit}\left(\frac{a}{L}\right)^2\,. \qquad (16)$$

It should be recognized that the small scale-length l results from the generation of the transverse currents in the midplane region, which are responsible for the steepening of the field line shape. In support of this, one can verify that the vanishing of l for $\beta = \beta_{crit}$ is attended formally with the field line fracture in the midplane, that is caused by the transverse current layer formation.

As follows from the discussion above, the solution of the equilibrium equation (3) exists if

$$\frac{dG}{dR} \geq 0\,. \qquad (17)$$

The last condition coincides with the well-known criterion of the mirror mode stability [3]. In paper [4] the requirement (17) was used for the definition of the maximum MHD-stable beta values in the numerical Fokker-Planck code evaluations. Besides, the condition (17) conforms with the results of paper [5], in which a well-posed equilibrium problem being discussed.

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