37. SU(3) ISOSCALAR FACTORS AND REPRESENTATION MATRICES

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The most commonly used SU(3) isoscalar factors, corresponding to the singlet, octet, and decuplet content of $8 \otimes 8$ and $10 \otimes 8$, are shown at the right. The notation uses particle names to identify the coefficients, so that the pattern of relative couplings may be seen at a glance. We illustrate the use of the coefficients below. See J.J de Swart, Rev. Mod. Phys. 35, 916 (1963) for detailed explanations and phase conventions.

A $\sqrt{-}$ is to be understood over every integer in the matrices; the exponent 1/2 on each matrix is a reminder of this. For example, the $\Xi \rightarrow \Omega K$ element of the $10 \rightarrow 10 \otimes 8$ matrix is $-\sqrt{6/24} = -1/2$.

Intramultiplet relative decay strengths may be read directly from the matrices. For example, in decuplet $\rightarrow$ octet + octet decays, the ratio of $\Omega^* \rightarrow \Xi K$ and $\Delta \rightarrow N \pi$ partial widths is, from the $10 \rightarrow 8 \times 8$ matrix,

$$\frac{\Gamma (\Omega^* \rightarrow \Xi K)}{\Gamma (\Delta \rightarrow N \pi)} = \frac{12}{6} \times \text{(phase space factors)}. \quad (37.1)$$

Including isospin Clebsch-Gordan coefficients, we obtain, e.g.,

$$\frac{\Gamma (\Omega^* - \rightarrow \Xi^0 K^-)}{\Gamma (\Delta^+ \rightarrow p \pi^0)} = \frac{1/2}{2/3} \times \frac{12}{6} \times p.s.f. = \frac{3}{2} \times p.s.f. \quad (37.2)$$

Partial widths for $8 \rightarrow 8 \otimes 8$ involve a linear superposition of $8_1$ (symmetric) and $8_2$ (antisymmetric) couplings. For example,

$$\Gamma (\Xi^* \rightarrow \Xi \pi) \sim \left( -\sqrt{\frac{9}{20}} g_1 + \sqrt{\frac{3}{12}} g_2 \right)^2. \quad (37.3)$$

The relations between $g_1$ and $g_2$ (with de Swart’s normalization) and the standard $D$ and $F$ couplings that appear in the interaction Lagrangian,

$$\mathcal{L} = -\sqrt{2} D Tr \left( \{ B, B \} M \right) + \sqrt{2} F Tr \left( [ B, B ] M \right), \quad (37.4)$$

where $[ B, B ] \equiv B B - B B$ and $\{ B, B \} \equiv B B + B B$, are

$$D = \sqrt{\frac{30}{40}} g_1, \quad F = \sqrt{\frac{6}{24}} g_2. \quad (37.5)$$

Thus, for example,

$$\Gamma (\Xi^* \rightarrow \Xi \pi) \sim (F - D)^2 \sim (1 - 2\alpha)^2, \quad (37.6)$$

where $\alpha \equiv F/(D + F)$. (This definition of $\alpha$ is de Swart’s. The alternative $D/(D + F)$, due to Gell-Mann, is also used.)

The generators of SU(3) transformations, $\lambda_a$ $(a = 1, 8)$, are $3 \times 3$ matrices that obey the following commutation and anticommutation relationships:

$$[ \lambda_a, \lambda_b ] \equiv \lambda_a \lambda_b - \lambda_b \lambda_a = 2i f_{abc} \lambda_c \quad (37.7)$$

$$\{ \lambda_a, \lambda_b \} \equiv \lambda_a \lambda_b + \lambda_b \lambda_a = \frac{4}{3} \delta_{ab} I + 2d_{abc} \lambda_c , \quad (37.8)$$

where $I$ is the $3 \times 3$ identity matrix, and $\delta_{ab}$ is the Kronecker delta symbol. The $f_{abc}$ are odd under the permutation of any pair of indices, while the $d_{abc}$ are even. The nonzero values are


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$1 \rightarrow 8 \otimes 8$

$(\Lambda) \rightarrow (N\overline{K} \Sigma \pi \Lambda \eta \Xi K) = \frac{1}{\sqrt{8}} (2 \ 3 \ -1 \ -2)^{1/2}$

$8_1 \rightarrow 8 \otimes 8$

$\left(\begin{array}{c}
N \\
\Sigma \\
\Lambda \\
\Xi \\
\end{array}\right) \rightarrow \left(\begin{array}{c}
N\pi N\eta \Sigma K \Lambda K \\
N\overline{K} \Sigma \pi \Lambda \pi \Sigma \eta \Xi K \\
N\overline{K} \Sigma \pi \Lambda \eta \Xi K \\
\Sigma K \Lambda \overline{K} \Xi \pi \Xi \eta \\
\end{array}\right) = \frac{1}{\sqrt{20}} \left(\begin{array}{c}
9 \\
-6 \\
2 \\
9 \\
\end{array}\right)$

$8_2 \rightarrow 8 \otimes 8$

$\left(\begin{array}{c}
N \\
\Sigma \\
\Lambda \\
\Xi \\
\end{array}\right) \rightarrow \left(\begin{array}{c}
N\pi N\eta \Sigma K \Lambda K \\
N\overline{K} \Sigma \pi \Lambda \pi \Sigma \eta \Xi K \\
N\overline{K} \Sigma \pi \Lambda \eta \Xi K \\
\Sigma K \Lambda \overline{K} \Xi \pi \Xi \eta \\
\end{array}\right) = \frac{1}{\sqrt{12}} \left(\begin{array}{c}
3 \\
2 \\
6 \\
3 \\
\end{array}\right)$

$10 \rightarrow 8 \otimes 8$

$\left(\begin{array}{c}
\Delta \\
\Sigma \\
\Xi \\
\Omega \\
\end{array}\right) \rightarrow \left(\begin{array}{c}
N\pi \Sigma K \\
N\overline{K} \Sigma \pi \Lambda \pi \Sigma \eta \Xi K \\
\Sigma K \Lambda \overline{K} \Xi \pi \Xi \eta \\
\Xi \overline{K} \\
\xi \overline{K} \\
\end{array}\right) = \frac{1}{\sqrt{12}} \left(\begin{array}{c}
-2 \\
-6 \\
3 \\
6 \\
-3 \\
12 \\
\end{array}\right)$

$8 \rightarrow 10 \otimes 8$

$\left(\begin{array}{c}
N \\
\Sigma \\
\Lambda \\
\Xi \\
\end{array}\right) \rightarrow \left(\begin{array}{c}
\Delta \overline{K} \Sigma \pi \Sigma \eta \Xi K \\
\Sigma \pi \Xi K \\
\Xi \pi \Xi \eta \Omega K \\
\end{array}\right) = \frac{1}{\sqrt{15}} \left(\begin{array}{c}
-12 \\
8 \\
-9 \\
3 \\
-3 \\
-3 \\
6 \\
\end{array}\right)$

$10 \rightarrow 10 \otimes 8$

$\left(\begin{array}{c}
\Delta \\
\Sigma \\
\Xi \\
\Omega \\
\end{array}\right) \rightarrow \left(\begin{array}{c}
\Delta \overline{K} \Sigma \pi \Sigma \eta \Xi K \\
\Sigma \pi \Xi K \\
\Xi \pi \Xi \eta \Omega K \\
\Xi \overline{K} \Omega \eta \\
\xi \overline{K} \Omega \eta \\
\end{array}\right) = \frac{1}{\sqrt{24}} \left(\begin{array}{c}
15 \\
8 \\
12 \\
-12 \\
-12 \\
\end{array}\right)$

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The $\lambda_a$'s are

$$
\begin{align*}
\lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\lambda_2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \\
\lambda_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \\
\lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
\lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \\
\lambda_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.
\end{align*}
$$

Equation (37.7) defines the Lie algebra of SU(3). A general $d$-dimensional representation is given by a set of $d \times d$ matrices satisfying Eq. (37.7) with the $f_{abc}$ given above. Equation (37.8) is specific to the defining 3-dimensional representation.