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E.V.Shuryak

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SUPPRESSION OF INSTANTONS AS THE ORIGIN OF CONFINEMENT

E.V.Shuryak

Institute of Nuclear Physics, Novosibirsk, USSR

Abstract. The strong enough quark and/or gluon fields suppress instantons and are therefore expelled from vacuum. Arguments are also given that instantons are really suppressed inside hadrons.

Since the discovery of instantons [1] it becomes clear that the vacuum structure of QCD is very complicated [2, 3]. Although instantons are shown to affect significantly the interaction of quarks and gluons [2, 3], the main problem of their confinement remains open.

Recently the important step in the phenomenology of vacuum structure has been done in [4], where the anomalous vacuum averages of fields have been introduced. The most important one, the square of the gauge field, has been found numerically from the charmonium sum rules [4]:

$$\langle vac | \frac{g^2}{4\pi^2} (G^a_{\mu\nu})^2 | vac \rangle \simeq 0.012 \;\; GeV^4 \qquad (1)$$

Instantons may provide such an effect, e.g. [2, 3, 4]:

$$\langle vac | \frac{g^2}{4\pi^2} (G^a_{\mu\nu})^2 | vac \rangle = 0.96 \int \frac{d\rho}{\rho^5} \left[\frac{8\pi^2}{g^2(\rho)} \right]^6 \exp\left[- \frac{8\pi^2}{g^2(\rho)} \right] \qquad (2)$$

where ρ is the instanton size, $8\pi^2/g^2(\rho) \equiv (11 - \frac{2}{3} n_f) \ln(1/\rho\mu)$ n_f is the number of flavors, $\mu \simeq m_\pi$. This integral rapidly diverges at large ρ, where the used dilute gas approximation fails, and in some unknown way (merons [3] ?) it is cut at $\rho \sim 1$ fermi, as is seen from (1) [4].

In this paper we show that at large enough quark density and/or field strength the integral over the instanton size is naturally cut off at some smaller value ρ_0, which significantly suppress the instanton effects like (1). The consequenses of this phenomenon is far going, since due to conformal anomaly of the trace of energy-momentum tensor $T_{\mu\mu}$ [5] (1) implies the negative vacuum energy density

$$\varepsilon \equiv \frac{1}{4}\langle vac|\,T_{\mu\mu}\,|vac\rangle = -\frac{(11-\frac{2}{3}n_f)g^2}{128\pi^2}\langle vac|(G^a_{\mu\nu})^2|vac\rangle \simeq -0.0035\ \mathrm{GeV}^4 \quad (3)$$

quite natural in the tunneling picture [3]. So, as far as quarks and gluons suppress instantons, they also suppress this energy gain. Therefore they are expelled from vacuum[1] and confined inside hadrons, being the bubbles of "normal" phase in the "abnormal" vacuum. We also present arguments, that instantons are really suppressed inside hadrons. The resulting picture is similar to MIT bag [6], but with rather different values of the involved parameters.

As the first example let us consider the cold quark plasma [7, 8], a medium of quarks with momenta $|\vec{p}\,| < p_F$. The polarization operator provides screening at Debye length $\rho_0 \sim (g(p_F)\cdot p_F)^{-1}$. Small ($\rho \ll \rho_c$) instantons are not affected, but for large ones the quantum fluctuations, normally giving ultraviolet log (ρ) and leading to $g(\rho)$ dependence, now are cut off at ρ_0. Therefore,

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[1] Note the close analogy to Meissner effect: magnetic field is expelled from superconductors for it suppresses Cooper pairing.

the tunneling probability $\exp(-8\pi^2/g^2(\rho_0))$ grows with ρ no more and the integral (2) is rapidly convergent. The resulting estimate is $(n_f = 3)$

$$\langle \text{plasma}\,|\,\frac{g^2}{4\pi^2}(G^a_{\mu\nu})^2\,|\text{plasma}\rangle \sim \left(\ln\left(\frac{p_F}{\mu}\right)\right)^{11}\cdot\mu^9\cdot p_F^{-5}, \quad p_F \gg \mu \qquad (4)$$

so, with increase in density such plasma rapidly becomes nearly "normal".

To make this estimate more accurate one has to find explicitely the instantons, deformed by plasma, and quantum fluctuation near it. We have no such solution at present, but are able to show that the action $8\pi^2/g^2$ remains practically unchanged. The reason is that the for massless quarks such plasma preserves the chiral invariance, so the instantons remains selfdual and $\frac{1}{4g^2}\int G^2 d^4x$ term in action is directly expressed via the topological charge. As for the quark loops, with the help of quark Green function [9] one can show that in such case they give zero correction.

For the related problem of "hot plasma" (nonzero temperature T and zero charge, discussed in [8, 10, 11]) the periodic instantons [12] may be used. All our conclusions remains valid with the obvious change $p_F \to T$.

The second problem is that about instantons in external field $G^{(ext)}_{a\mu\nu}$, not extended to infinity so that the topological charge can easily be defined. All arguments are close to those above, the role of the cut off length ρ_0 is now played by the curvature of the trajectories in it, $\rho_0 \sim (g\,G^{ext})^{-1/2}$. The fact, that in strong enough field the ultraviolet logs are of this kind is known, see e.g. the Heisenberg-Euler effective action of QED.

Again, the explicite solution for $\rho \gtrsim \rho_0$ is absent and all we can say is that for selfdual $G^{(ext)}_{a\mu\nu}$ the action is unchanged. This is also seen from the weak field result [3]

$$\delta S = \frac{\pi^4 \rho^4}{6g^2}\left(G^{(ext)}_{a\mu\nu} - \tilde{G}^{(ext)}_{a\mu\nu}\right)^2, \quad \tilde{G}_{\mu\nu} \equiv \tfrac{1}{2}\epsilon_{\mu\nu\sigma\tau} G_{\sigma\tau} \qquad (5)$$

Note, that the sign of this correction means the instanton stimulation rather than their suppression, which is one more manifestation of the instability of "normal" phase with zero or small $(G^{ext} \lesssim \sqrt{\langle vac|G^2|vac\rangle} \sim \mu^2)$ field.

Finally we come to brief discussion of hadrons. Let us take the $T_{\mu\mu}$ average over the one-nucleon state

$$m_N \bar{\Psi}_N \Psi_N = \langle N|T_{\mu\mu}|N\rangle - \langle vac|T_{\mu\mu}|vac\rangle \qquad (6)$$

Due to the small light quark mass, their contribution to $T_{\mu\mu} = \sum_i m_i \bar{\Psi}_i \Psi_i$ is small $[13]^{2)}$ and the nucleon mass can be considered as due entirely to $\langle G^2\rangle$ difference in vacuum and inside the nucleon. Taking the typical nucleon volume $V_N \simeq 1(\text{fermi})^3$ one has the left hand part of (6) $m_N/V_N \simeq 0.01$ GeV^4, which is quite in agreement with the last term $\langle vac|T_{\mu\mu}|vac\rangle$ (3). So, the $\langle N|G^2|N\rangle$ term is really smaller, the nucleon is "normal" inside. Note, that μ/ρ_F is hot really small in this case, $\sim 1/2$, but the very strong dependence (4) explains this observation.

This picture is very close to MIT bag model [6], in terms of which we say that the volume energy $B \cdot V_N = \frac{1}{4} m_N$; and that B is just the vacuum energy (3) with another sign. The first conclusion well agrees with [6], but the values of B, V_N disagrees: $B \simeq 5 \cdot 10^{-4}$ GeV^4 [6] and V_N = 4 +.5 (fermi)^3, the unrealistically large volume where "the quark wave function differs from zero". This difference is important for barion gas - quark plasma transition [7, 8] and for the determination of the upper limit of the stable star mass. Another application is hadronic collisions, which produce the "normal" excited system, expanding against the "abnormal" vacuum pressure.

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[2)] If one takes T_{00} instead of $T_{\mu\mu}$, the quark term is dominant and "the nucleon is made up of quarks".

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