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A NEW THRESHOLD FOR PHOTOELECTRIC EFFECT IN HYDROGEN ATOM

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A b s t r a c t

We present new data which clearly predict a strong ionization of hydrogen atom in a monochromatic field at frequencies above a chaotic threshold which is much below the conventional one-photon threshold. This effect, which leads to a much stronger ionization than the usual photoelectric effect, is a quantum manifestation of the classical chaos and can be tested in laboratory experiments.

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The cross-section for one-photon ionization in hydrogen atom is well known and can be calculated according to elementary quantum mechanics. For frequencies below the ionization threshold, two or more photons are required and, for sufficiently small fields, the probability of such processes is much smaller than the standard one-photon ionization.

On the other hand, previous works [1-8] have indicated that strong excitation and ionization can take place even for frequencies well below the one photon ionization. This quantum phenomenon is connected with the appearence of chaotic motion in the corresponding classical system which leads to a diffusion process describing by Fokker-Planck-Kolmogorov equation.

In order to clarify the above situation we analyze here the ionization mechanism for a wide range of frequencies. To this end we consider a hydrogen atom under a linearly polarized monochromatic electric field in initial state with principal quantum number $n_o \gg 1$. For simplicity we restrict ourselves to the study of very extended states with parabolic quantum numbers $n_1 = n_o - 1$, $n_2 = 0$, and magnetic quantum number $m = 0$. For such states it is possible to use a one-dimensional approximation and to describe the ionization process by the one-dimensional Hamiltonian:

$$H = P^2/2 - 1/x + \varepsilon x \cos(\omega t); \quad x > 0 \qquad (1)$$

where ε and ω are the field strength and frequency in atomic units. The validity of this one-dimensional approximation is due to the small value of matrix elements for the transitions with $\Delta n_2 \neq 0$. As a consequence the atom remains one-dimensional during the interaction time [3,4]. This important fact has been also checked in laboratory experiments in which such states were reproduced and excited by microwaves fields [9].

Classical analysis [2,10] shows that for field strengths $\varepsilon_o = \varepsilon n_o^4$ higher than a critical value

$$\varepsilon_c \approx 1/(50\,\omega_o^{1/3}) \qquad (2)$$

where $\omega_o = \omega n_o^3$ is assumed to be $\gtrsim 1$ and n_o is the initially excited state, the chaotic excitation of the electron takes place according to the diffusion law. The diffusion rate is given by

$$D = \frac{<(\Delta n)^2>}{\tau} \approx 2\varepsilon_o^2 n^3/(\omega_o^{7/3} n_o) = 2\varepsilon^2 n^3/\omega^{7/3} \qquad (3)$$

where $\tau = \omega t/2\pi$ is time measured in the number of microwave periods.

Due to the rapid growth of the diffusion coefficient with n, ionization takes place after a finite time τ_d which can be estimated [2] from the condition $\Delta n \sim n_o$:

$$\tau_d \sim \omega_o^{7/3}/\varepsilon_o^2 \qquad (4)$$

Since the classical diffusion is due to resonances between the electric field and the harmonics of the electron motion, it takes place only for frequencies

$$\omega_o > \omega_c \sim 1 \qquad (5)$$

where ω_c is a critical frequency that we will call chaotic threshold. Below the chaotic threshold, Eq.(2) is no longer valid and transition to classical chaos would require a considerably larger field close to the static threshold $\varepsilon_s \approx 0.13$. Notice that threshold (5) holds only if $\varepsilon_o \gtrsim 1/50$. Otherwise, the chaotic threshold has to be determined from Eq.(2), and is equal $\omega_c \approx (50\ \varepsilon_o)^3$.

A recent analysis [4] of the quantum behaviour has shown the existence of a critical field value ε_q, the quantum delocalization border, below which quantum effects suppress the diffusion excitation. However, for field values above this border, the quantum excitation goes approximately in the same way as the classical one. This critical value is given by [4]

$$\varepsilon_q \approx \omega_o^{7/6}/\sqrt{6 n_o} \qquad (\omega_o \gtrsim 1) \qquad (6)$$

Notice that for levels $n_o \lesssim 400\ \omega_o^3$, $\varepsilon_q > \varepsilon_c$ and the condition of diffusion excitation is determined by the quantum delocalization border (6).

It is interesting to compare the diffusive ionization with the standard one-photon process. According to ref. [11] the rate of latter process is

$$\tau_\phi^{-1} \approx 1.7\ \varepsilon_o^2 n_o^2/\omega_o^{13/3} \qquad (7)$$

This rate reaches its maximum $\gamma_\phi \approx 34\,\varepsilon_o^2/n_o^{7/3}$ at the frequency threshold value $\omega_o = \omega_\phi \approx n_o/2$. On the other hand the diffusive ioniziation rate τ_d^{-1} reaches its maximum $\gamma_d \sim \varepsilon_o^2$ at $\omega_o \approx \omega_c \ll \omega_\phi$.

The striking result is that not only the diffusive ionization occurs at very low frequencies as compared to the one-photon threshold, but also its rate is much higher:

$$\gamma_d/\gamma_\phi \sim n_o^{7/3}/34 \qquad (8)$$

The above rates are measured in number of periods of external field. In terms of the real physical time $t = (2\pi/\omega)\tau$ the ratio becomes

$$\Gamma_d/\Gamma_\phi \sim n_o^{4/3}/17 \qquad (9)$$

From these estimates we see that, for high levels, the diffusive ionization is much more effective than the direct one-photon transition.

We checked the above predictions by a numerical solution of the Schroedinger equation for the model (1). In these computations we used the Sturm basis in order to explicitly take into account the continuous part of the spectrum. A detailed description of our numerical technique will be given elsewhere [8].

In accordance with laboratory experiments we define ionization W_I as the excitation above a sufficiently large unperturbed level $n = \bar{n}$. In Figs. 1 and 2 we present our main results on the comparison between the diffusive and one-photon ionization mechanisms for fixed field intensity ε_o^2, initial state n_o, and physical time t (full lines). The much higher efficiency of diffusive ionization is striking. Moreover the frequency ionization threshold is down by almost two-orders of magnitude!

The dotted lines are obtained by numerically integrating the Newton equations of motion for 250 trajectories with the same initial action n_o and phases homogeneously distributed within the interval $[0, 2\pi]$ thus corresponding to the same initial quantum state.

The new frequency threshold ω_c for photoelectric effect is determined by the classical chaos border. Numerically, this threshold turns out to be lower than unity due to overlap of high order resonances in a strong field as well as to a finite width of the primary resonance $\omega_o = 1$.

As is seen from Fig. 1, in the range $\omega_c \leq \omega_o \leq \omega_\ell \approx (6 n_o \mathcal{E}_o^2)^{3/7}$ (see Eq.(6)) both classical and quantum behaviour are close as a result of the quantum delocalization phenomenon. For $\omega_o > \omega_\ell$ quantum localization occurs [4] and ionization probability sharply drops as compared to the classical behaviour. In Fig.2, the field value $\mathcal{E}_o = 0.075$ just above the delocalization border (at $\omega_o = 1$), and the lower initially excited level $n_o = 30$ yields a slightly more complicated behaviour whose details will be discussed elsewhere [8].

In the one-photon region, $\omega_o > \omega_\phi$ numerical results are in good agreement with the theory (Fig. 3). This is a check of our numerical procedure which shows that the Sturm basis efficiently takes into account the continuous spectrum. Another check is given in Fig. 4 as the dependence of ionization probability on $\mathcal{E}_o$ and τ.

As an additional evidence that we are here in presence of a new phenomenon, which cannot be explained by ordinary multiphoton ionization, we show in Fig. 5 the ionization probability vs. field strength for $\omega_o = 1$, $n_o = 66$. For ordinary multiphoton ionization, the dependence would be $W_I \propto \mathcal{E}_o^{2k}$ where $k = 19$ is equal to the number of photons required for ionization. On the contrary, numerical results give $k \approx 7$ only.

The present high level of experimental art [5, 9, 12, 13] makes it quite possible to observe the described phenomena in laboratory.

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FIGURE CAPTIONS

Fig. 1 Ionization probability $W_I = \sum_{n>\bar{n}} |C_n|^2$ versus field frequency ω_o after a time $\tau = 40\omega_o$ which corresponds to the same real physical time t for all frequencies. Here, $n_o = 66$, $\varepsilon_o = .05$, $\bar{n} = 99$. (x) Quantum case, (o) Classical case. Here and below the logarithms are decimal.

Fig. 2 Same as Fig. 1 with $n_o = 30$, $\varepsilon_o = .075$, $\bar{n} = 90$.

Fig. 3 Ionization probability for frequencies above the one-photon threshold ω_ϕ for the case of Fig. 2. The straight line is the theoretical expression. The crosses are the results of numerical computations. The excellent agreement with the theory even for very large frequencies is a check of our numerical computations and shows that the Sturm basis efficiently takes into account the continuous spectrum.

Fig. 4 Ionization rate vs. field intensity for the case $n_o = 30$, $\omega_o = 30$ (Fig. 4a), and ionization probability vs. time (4b) for the case $\omega_o = 30$, $\varepsilon_o = 0.075$. The straight lines are drawn according to the theoretical expression and the crosses are results of numerical computations. Here also notice a very good agreement between theory and numerical results.

Fig. 5 Ionization probability vs. field intensity. Here $n_o = 66$, $\omega_o = 1$, $\tau = 60$, $\bar{n} = 99$. Fitting the numerical data with the theoretical expression $W_I = C\ \varepsilon_o^{2k}$ (straight line) we obtain $k \approx 7$ which is much less than the value $k = 19$ for the ordinary multiphoton ionization.

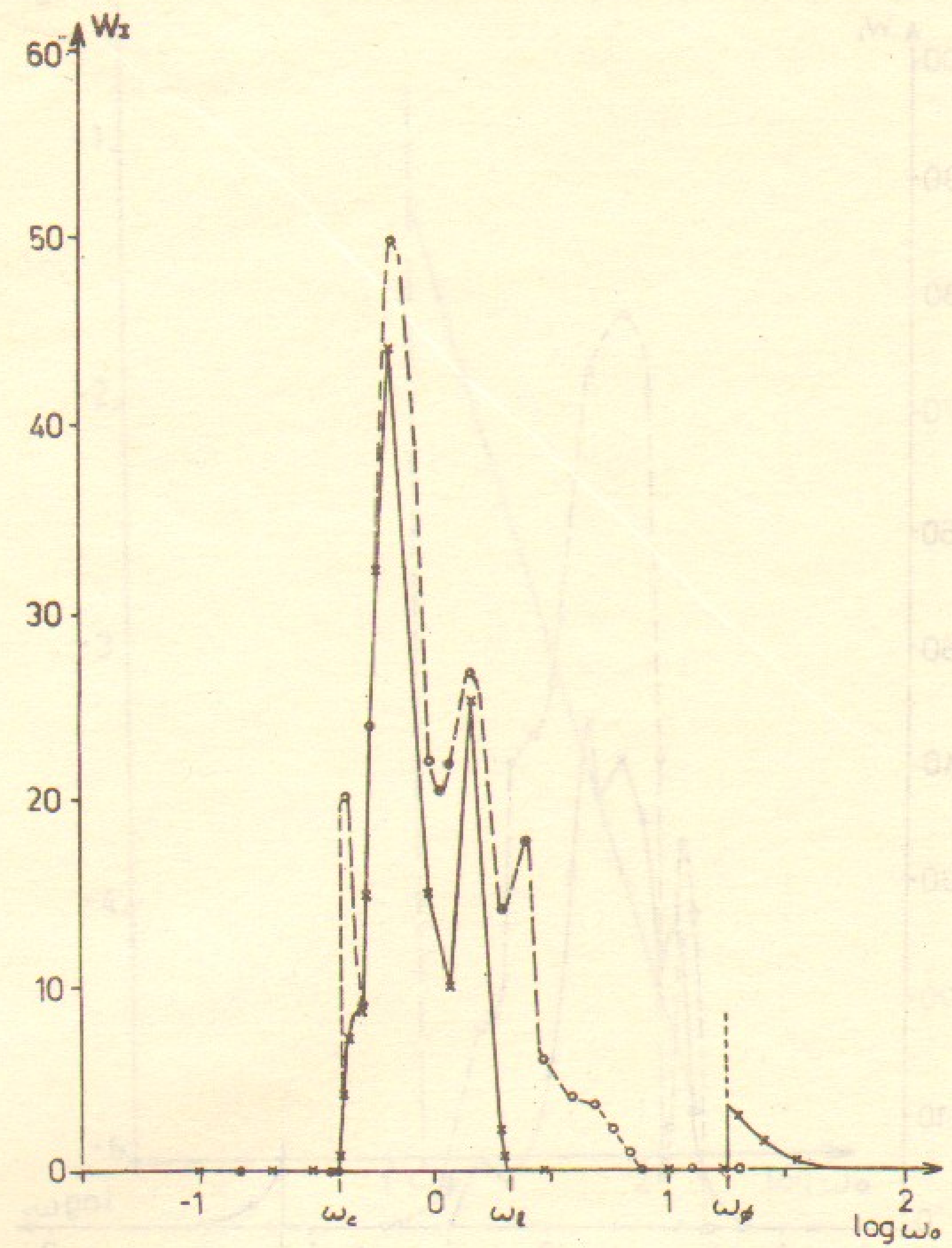
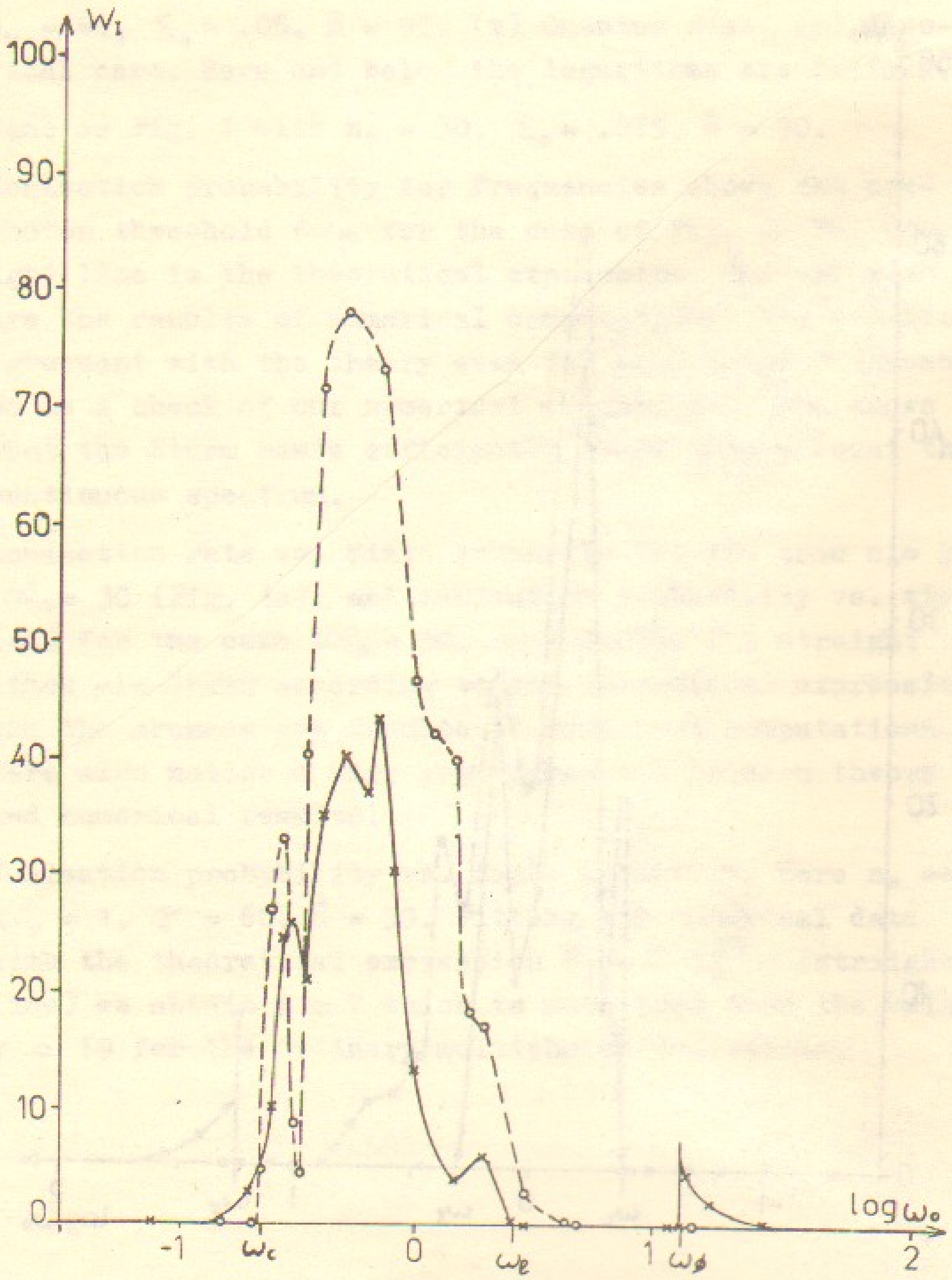
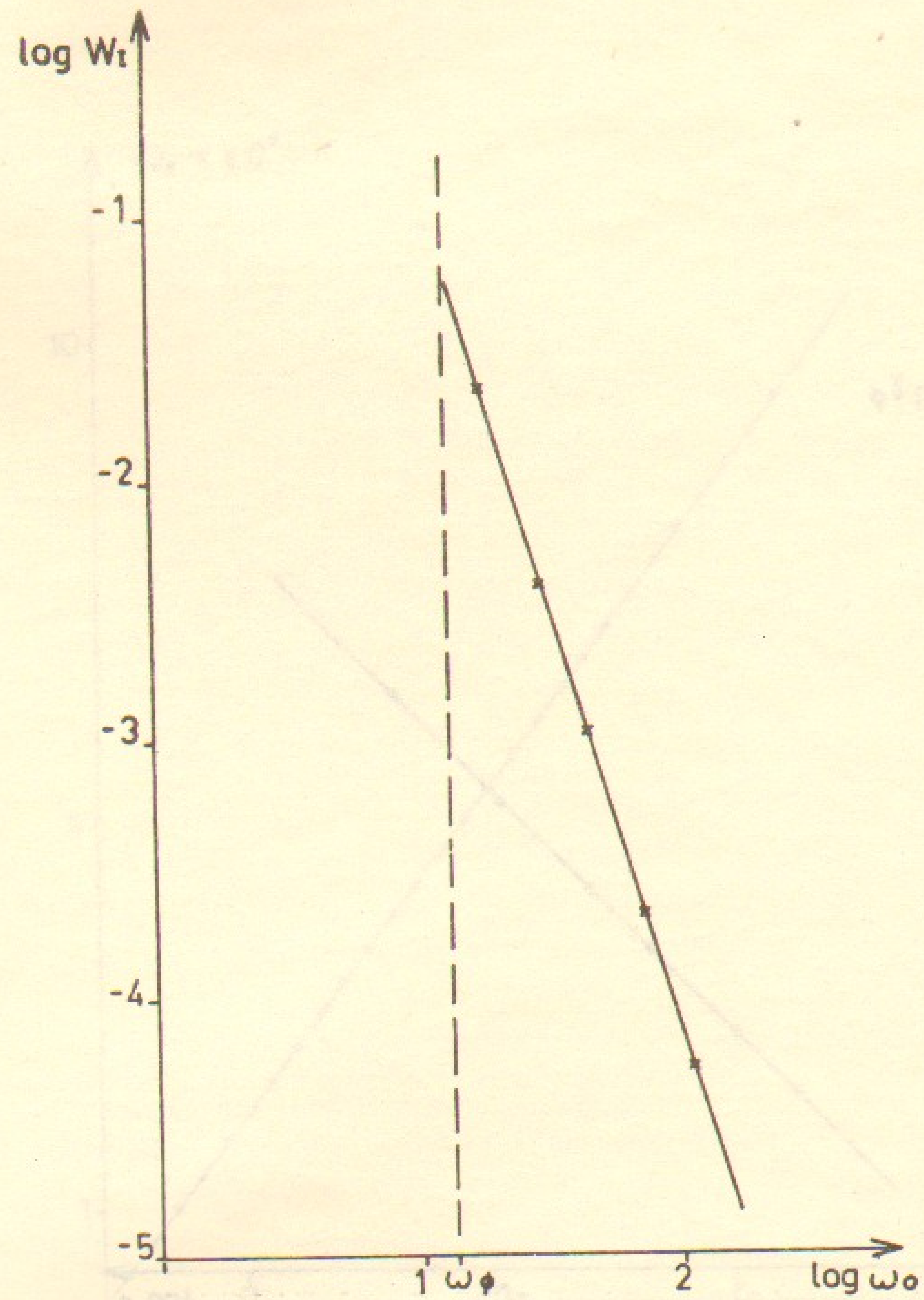


Fig.1



W_I
100
90
80
70
60
50
40
30
20
10
0
-1
0
1
2
$\log \omega_o$
ω_c
ω_ℓ
ω_ϕ

Fig.2



$\log W_I$
-1
-2
-3
-4
-5
1
ω_ϕ
2
$\log \omega_o$

Fig.3

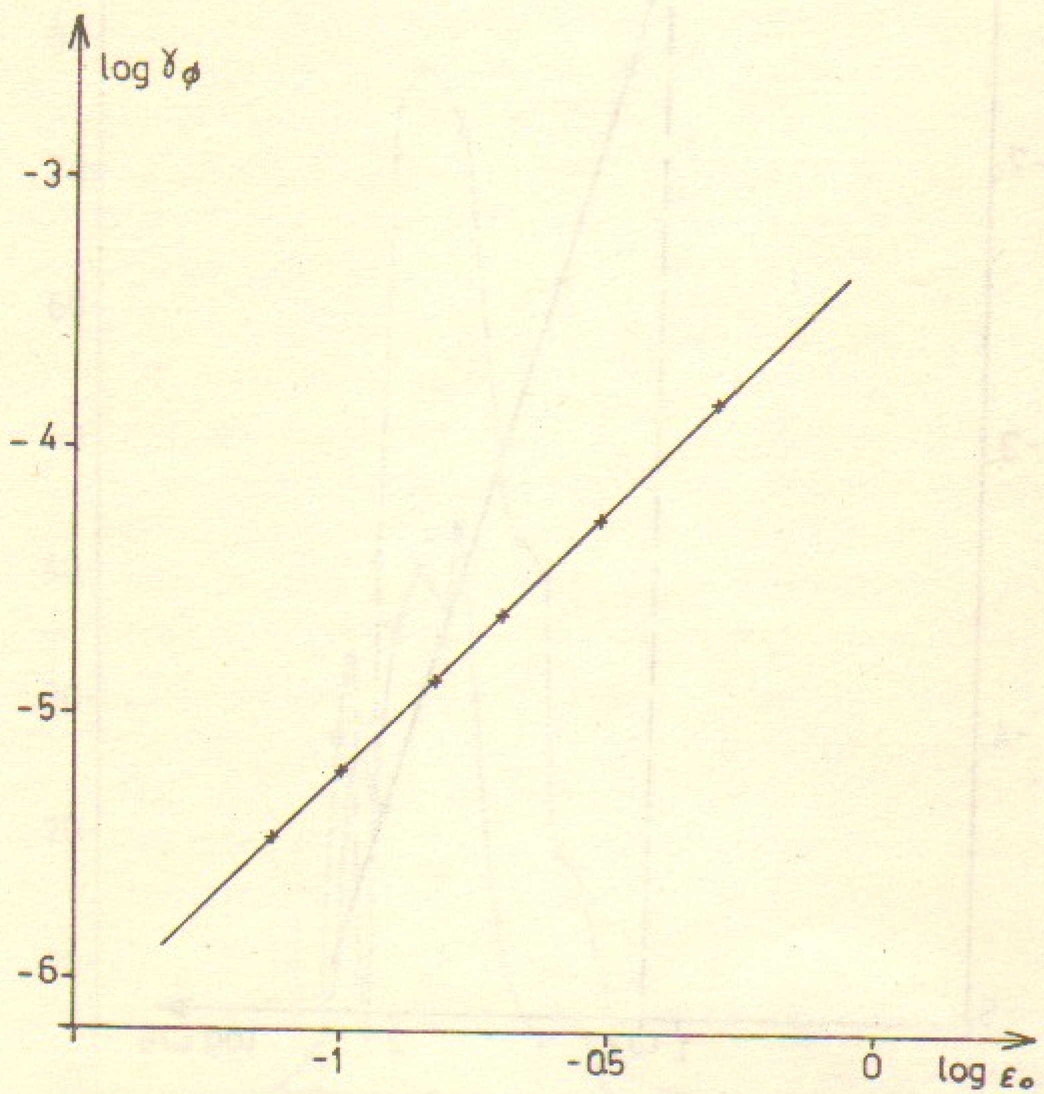


Fig. 4a

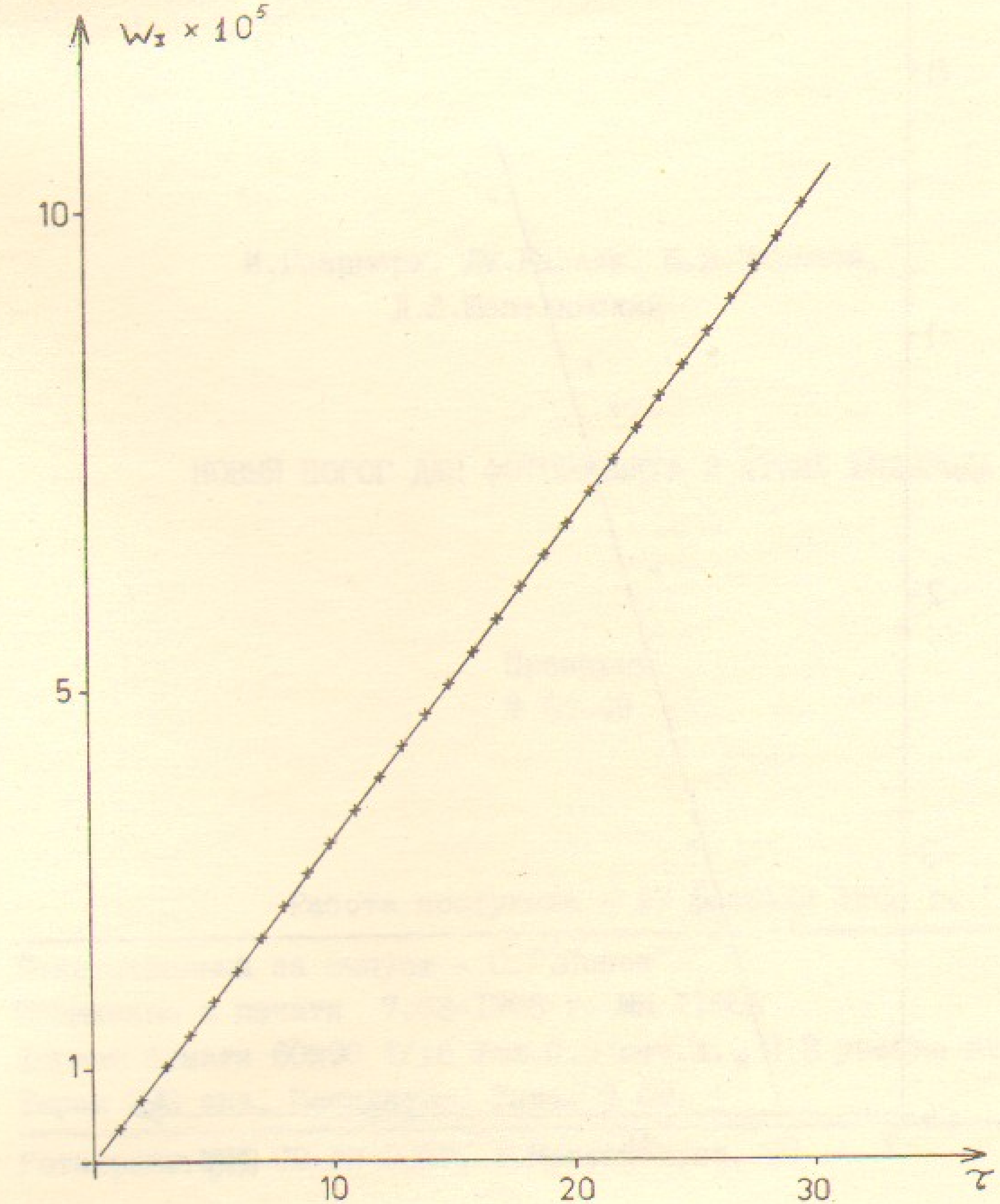


Fig. 4b

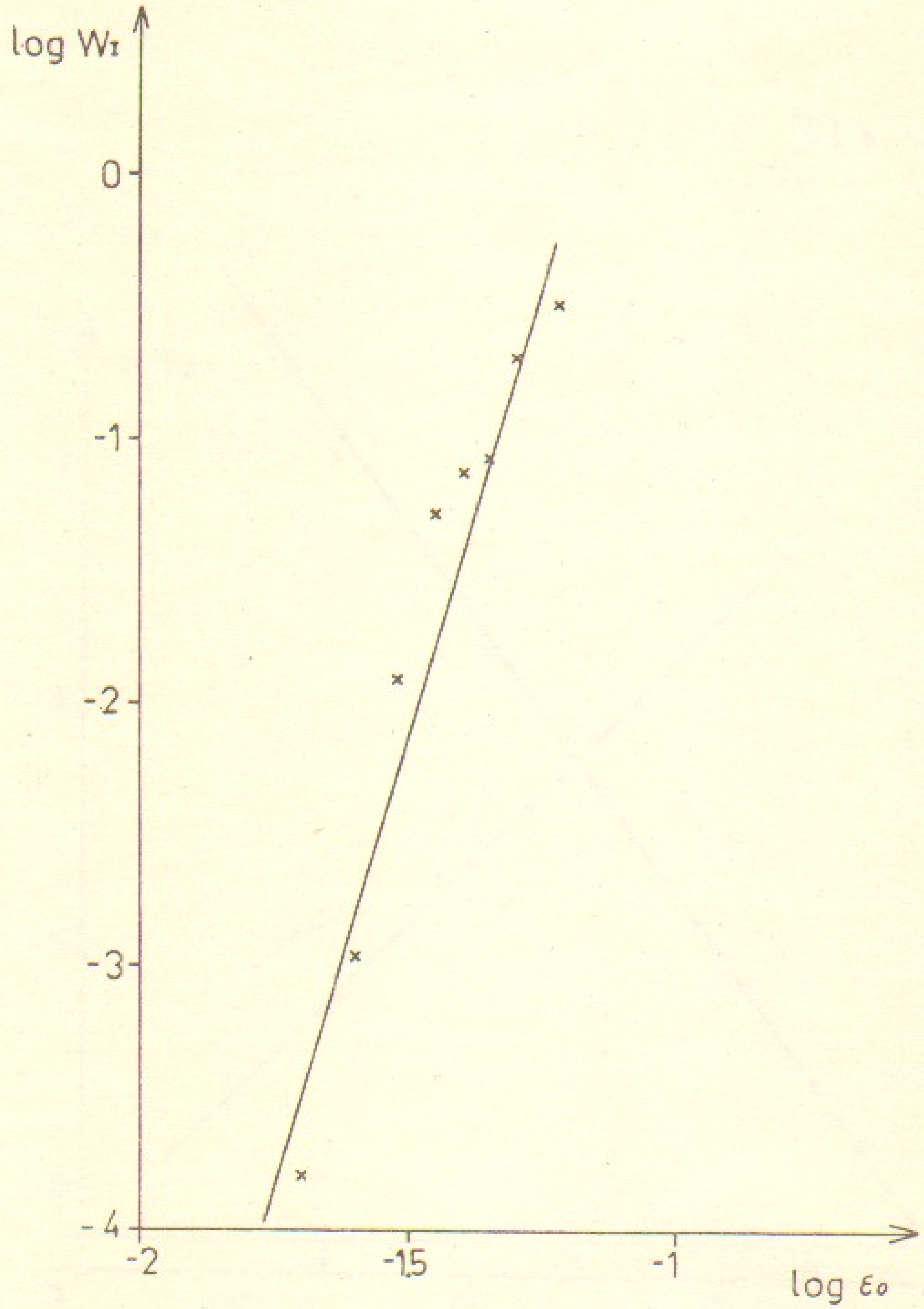


Fig.5

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НОВЫЙ ПОРОГ ДЛЯ ФОТОЭФФЕКТА В АТОМЕ ВОДОРОДА

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