

K. 93  
19.97

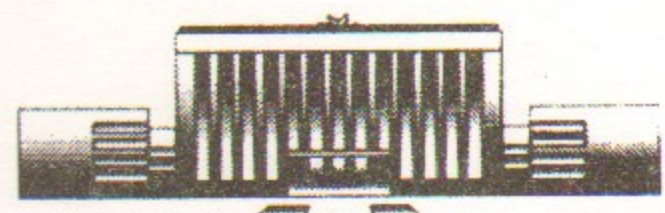
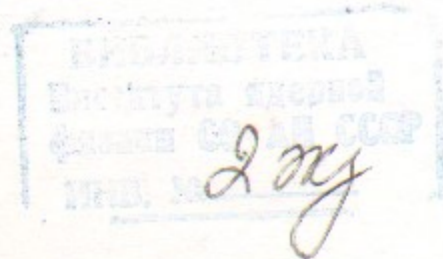


Budker Institute of Nuclear Physics  
SB RAS

L.M. Kurdadze, Z.K. Silagadze

ON SOME RARE WEAK DECAYS  
OF VECTOR MESONS

Budker INP 97-101



Novosibirsk

V

## On Some rare weak decays of vector mesons

L.M. Kurdadze, Z.K. Silagadze

Budker Institute of Nuclear Physics SB RAS  
630090 Novosibirsk, Russia

### Abstract

Some semileptonic weak decays of vector mesons are considered in the framework of the most popular quark models. The predicted branching ratios are unfortunately too small to make a study of these decays realistic at meson factories under construction.

## 1 Introduction

Weak decays of hadrons play an important role in our understanding of both perturbative and nonperturbative aspects of Standard Model. On the one hand they involve Kobayashi-Maskawa matrix elements and higher order corrections to weak currents. The latter are calculable perturbatively to high accuracy within the Standard Model framework, and the former are crucial parameters of the theory, not determined by it, but extracted from experiments. On the contrary, another ingredient of these weak decays, hadronic matrix elements of the weak currents are not calculable at present from the first principles and are subject of nonperturbative QCD, the acronym which in reality means a paradise for various phenomenological models of hadron structure.

Semileptonic decays with  $0^- \rightarrow 0^-$  and  $0^- \rightarrow 1^-$  hadron transitions attracted considerable attention, as they are promising experimental sources for extracting the Kobayashi-Maskawa matrix elements. The reviews of the theoretical models, involved in such a type of exercise, along with relevant literature can be found in [1-4] and we don't repeat them here. Instead we focus our efforts on giving a reliable estimate for semileptonic decays with  $1^- \rightarrow 0^-$  hadron transitions. Such weak decays escaped consideration simply because very tiny rates are expected for them. Indeed, rough estimate of the semileptonic decays rate is given by the one third of the free quark decay width, assuming that the spectator antiquark is irrelevant. It is straightforward to get this decay width [5]

$$\Gamma(Q \rightarrow qe\bar{\nu}) = \frac{G_F^2 m_Q^5}{192\pi^3} |V_{qQ}|^2 F\left(\frac{m_q}{m_Q}\right), \quad (1)$$

where  $V_{qQ}$  is the relevant Kobayashi-Maskawa matrix element and  $F(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x$  is a phase space factor, lepton mass being neglected.

Branching ratio, which follows from (1), for example, for  $J/\Psi \rightarrow D_s^- e^+ \nu$  decay is about  $10^{-9}$ , and other vector meson weak semileptonic decay branching ratios appear to be even smaller.

Below we use two the most popular models to give more elaborate estimates for the vector meson semileptonic decay rates. These calculations were motivated by the fact that several high luminosity meson factories are expected to come into operation in near future.

## 2 General Considerations

Let us consider  $V(Q\bar{Q}) \rightarrow P(q\bar{Q})e^- \bar{\nu}$  semileptonic decay, where  $V(Q\bar{Q})$  and  $P(q\bar{Q})$  stand for vector  $1^-$  and pseudoscalar  $0^-$  mesons, made up from  $Q\bar{Q}$  and  $q\bar{Q}$  quark-antiquark pairs, respectively. Corresponding amplitude looks like

$$A = \frac{G_F}{\sqrt{2}} V_{qQ} \bar{u}_e(\vec{k}_1) \gamma^\mu (1 - \gamma_5) v_\nu(\vec{k}_2) \langle P | J_\mu(0) | V \rangle$$

and we will have after averaging over the vector meson polarization and summing over the leptons spins (lepton mass is neglected and  $u^+ u = 2E$  normalization is used for lepton spinors)

$$|A|^2 = \frac{1}{3} G_F^2 |V_{qQ}|^2 S p(1 - \gamma_5) \hat{k}_1 \gamma^\mu \hat{k}_2 \gamma^\nu \sum_{\nu} \langle V | J_\nu^+(0) | P \rangle \langle P | J_\mu(0) | V \rangle \quad (2)$$

Let us decompose [6]

$$\sum_{\nu} \langle V(P, \epsilon) | J_\nu^+(0) | P(P') \rangle \langle P(P') | J_\mu(0) | V(P, \epsilon) \rangle = -\alpha g_{\mu\nu} + \sum_{\sigma_1, \sigma_2 = \pm} \beta_{\sigma_1 \sigma_2} (P + \sigma_1 P')_\mu (P + \sigma_2 P')_\nu + i \gamma \epsilon_{\mu\nu\lambda\sigma} (P + P')^\lambda (P - P')^\sigma \quad (3)$$

and note that terms from (3) containing  $(P - P')_\mu$  or  $(P - P')_\nu$  don't contribute to (2), because, for example

$$(P - P')_\mu S p(1 - \gamma_5) \hat{k}_1 \gamma^\mu \hat{k}_2 \gamma^\nu = S p(1 - \gamma_5) \hat{k}_1 (\hat{k}_1 + \hat{k}_2) \hat{k}_2 \gamma^\nu = 0$$

as lepton mass is assumed to be zero.

So only  $\alpha, \beta_{++}$  and  $\gamma$  invariant form-factors contribute to  $|A|^2$  and it is straightforward to get the following expression for the differential width [6, 7] (in  $\vec{P} = 0$  vector meson rest frame)

$$\frac{d^2 \Gamma(V \rightarrow P e \bar{\nu})}{dx dy} = \frac{1}{3} \frac{G_F^2 M_V^5}{32 \pi^3} |V_{qQ}|^2 \left\{ \alpha \frac{y}{M_V^2} + 2\beta_{++} [4x(x_+ - x) - y(1 - 2x)] - 2\gamma y(x_+ - 2x + \frac{1}{2}y) \right\} \quad (4)$$

where  $x_+ = \frac{1}{2}(1 - \frac{M_P^2}{M_V^2})$  and we have introduced dimensionless variables  $x = E_e/M_V$  and  $y = \frac{(P - P')^2}{M_V^2} \equiv \frac{t}{M_V^2}$ ,  $E_e$  being the lepton energy.

Thus decay width

$$\Gamma(V \rightarrow P e \bar{\nu}) = \int_{x_-}^{x_+} \int_{y_-}^{y_+} \frac{d^2 \Gamma}{dx dy} \quad (5)$$

with  $(x_+)$  was given above) [7]

$$x_- = 0, y_- = 0, y_+ = \frac{4x(x_+ - x)}{1 - 2x}$$

These integration limits are determined by decay kinematics.

Note that for decays to  $e^+ \nu$  the sign of the term proportional to  $\gamma$  in (4) should be reversed. The simplest way to see this is the following. If lepton mass is neglected, when  $\frac{d^2 \Gamma(V \rightarrow P^- e^+ \nu)}{dx dy}$  can be obtained from  $\frac{d^2 \Gamma(V \rightarrow P^+ e^- \bar{\nu})}{dx dy}$  by replacement  $x \rightarrow x^* = \frac{E_\nu}{M_V}$ . But it is easy to see that  $x^* = x_+ - x + \frac{1}{2}y$  and so  $4x^*(x_+ - x^*) - y(1 - 2x^*) = 4x(x_+ - x) - y(1 - 2x)$ , but  $x_+ - 2x^* + \frac{1}{2}y = -(x_+ - 2x + \frac{1}{2}y)$ .

It is convenient to introduce form-factors which characterize hadronic matrix element itself

$$\langle P(P') | J_\mu(0) | V(P, \epsilon) \rangle = i g \epsilon_{\mu\nu\lambda\sigma} \epsilon^\nu (P + P')^\lambda (P - P')^\sigma - f \epsilon_\mu - (\epsilon \cdot P') a_+ (P + P')_\mu - (\epsilon \cdot P') a_- (P - P')_\mu \quad (6)$$

Comparing (6) and (3), and using

$$\sum_{s_V} \epsilon_\mu \epsilon_\nu^* = -g_{\mu\nu} + \frac{P_\mu P_\nu}{M_V^2},$$

it is easy to find

$$\begin{aligned} \alpha &= f^2 + 4M_V^2 g^2 \bar{P}'^2, \quad \gamma = 2gf \\ \beta_{++} &= \frac{f^2}{4M_V^2} - M_V^2 g^2 y + \frac{1}{2} \left[ \frac{M_P^2}{M_V^2} - y - 1 \right] f a_+ + a_+^2 \bar{P}'^2, \end{aligned} \quad (7)$$

where

$$\bar{P}'^2 = \frac{[M_V^2(1-y) + M_P^2]^2}{4M_V^2} - M_P^2.$$

Another popular set of form-factors is defined through [8]

$$\begin{aligned} \langle P(P') | J_\mu(0) | V(P, \epsilon) \rangle &= \frac{2i}{M_V + M_P} \epsilon_{\mu\nu\lambda\sigma} \epsilon^\nu P'^\lambda P^\sigma V(q^2) - \\ &- \epsilon_\mu (M_V + M_P) A_1(q^2) - \frac{\epsilon \cdot q}{M_V + M_P} (P + P')_\mu A_2(q^2) + \dots, \quad q = P - P'. \end{aligned} \quad (8)$$

Dots here is for terms proportional to  $(P - P')_\mu$ , which do not contribute to decay width for massless lepton.

Obvious relations between these two sets of form-factors are

$$g(q^2) = \frac{V(q^2)}{M_V + M_P}, \quad f(q^2) = (M_V + M_P) A_1(q^2), \quad (9)$$

$$a_+(q^2) = -\frac{A_2(q^2)}{M_V + M_P}.$$

Some model for hadron structure is needed to concretize the introduced form-factors.

### 3 ISGW model

The Isgur-Scora-Grinstein-Wise model [9] uses nonrelativistic quark model wave functions to predict weak hadronic form-factors. Strictly speaking, this model becomes rigorous in the weak binding limit where  $M_V \approx 2m_Q$  and  $M_P \approx m_Q + m_q$ , and near the zero recoil point where  $t = q^2$  reaches its maximum value  $t_m = (M_V - M_P)^2$ . But it is assumed that the resulting

form-factor formulas are valid even beyond the weak binding regime. More serious problem is that the nonrelativistic quark model predictions about the  $(t_m - t)$ -dependence of form-factors are not reliable when  $t_m - t$  becomes large compared to typical hadronic scales. Nevertheless this model proved to be successful and up to now remains one the most popular one, maybe because "it is better to have the right degrees of freedom moving at the wrong speed than the wrong degrees of freedom moving at right speed" [10]. Updated version of the ISGW model, which incorporates relativistic corrections, heavy quark symmetry constraints and more realistic behavior of form-factors at large  $t_m - t$ , is given in [10].

In the weak binding limit the state vectors of the nonrelativistic  $V(Q\bar{Q})$  vector or  $P(q\bar{Q})$  pseudoscalar mesons can be represented as a superposition of the free quark-antiquark states [9, 11]

$$\begin{aligned} |V(\vec{P}, \epsilon)\rangle &= \sqrt{2M_V} \int \frac{d\vec{p}}{(2\pi)^3} \sum_{ms\bar{s}} C_{s\bar{s}}^{1m} \times \\ &\times \epsilon \cdot \epsilon_{(m)}^* \varphi_V(\vec{P}) |Q\left[\frac{m_Q}{M_V} \vec{P} + \vec{p}, s\right] \bar{Q}\left[\frac{m_Q}{M_V} \vec{P} - \vec{p}, \bar{s}\right]\rangle, \\ |P(\vec{P})\rangle &= \sqrt{2M_P} \int \frac{d\vec{p}}{(2\pi)^3} \sum_{ms\bar{s}} C_{s\bar{s}}^{00} \times \\ &\times \varphi_P(\vec{P}) |q\left[\frac{m_q}{M_P} \vec{P} + \vec{p}, s\right] \bar{Q}\left[\frac{m_Q}{M_P} \vec{P} - \vec{p}, \bar{s}\right]\rangle. \end{aligned} \quad (10)$$

We use  $\langle \vec{P}' | \vec{P} \rangle = (2\pi)^3 2E \delta(\vec{P}' - \vec{P})$  normalization for the meson state vectors while  $\langle \vec{p}' | \vec{p} \rangle = (2\pi)^3 \frac{E}{m} \delta(\vec{p}' - \vec{p})$  one for the state vectors of quark (or antiquark) with mass  $m$ .  $\epsilon_{(-)}$ ,  $\epsilon_{(0)}$  and  $\epsilon_{(+)}$  are three independent polarization 4-vectors for the vector mesons.  $C_{s\bar{s}}^{jj_s}$  are the usual Clebsh-Gordan coefficients that couple  $s$  and  $\bar{s}$  quark and antiquark spins into the meson spin and polarization. At last,  $|Q[\vec{p}_1, s] \bar{Q}[\vec{p}_2, \bar{s}]\rangle = a_s^+(\vec{p}_1) b_{\bar{s}}^+(\vec{p}_2) |0\rangle$ ,  $a^+$  and  $b^+$  being the quark and antiquark creation operators. Note that our normalization convention indicates the following anticommutation relations

$$\{a_s(\vec{p}), a_s^+(\vec{p}')\} = \{b_s(\vec{p}), b_s^+(\vec{p}')\} = (2\pi)^3 \frac{E}{m} \delta(\vec{p}' - \vec{p}). \quad (11)$$

To obtain the quark model weak transition matrix element, one should replace  $J_\mu(0)$  weak current in  $\langle P(P') | J_\mu(0) | V(P, \epsilon) \rangle$  by the quark weak current  $j_\mu(0) = \bar{q}(0) \gamma_\mu (1 - \gamma_5) Q(0)$  (the Kobayashi-Maskawa matrix element was

already separated), decompose quark field operator (note  $u^{+(\lambda)}(\vec{k})u^{+(\lambda')}(\vec{k}) = v^{+(\lambda)}(\vec{k})v^{+(\lambda')}(\vec{k}) = \frac{k_0}{m}\delta_{\lambda\lambda'}$ , normalization for the Dirac spinors)

$$\Psi(0) = \sum_{\lambda} \int \frac{d\vec{k}}{(2\pi)^3} \frac{m}{k_0} [a_{\lambda}(\vec{k})u^{(\lambda)}(\vec{k}) + b_{\lambda}^+(\vec{k})v^{(\lambda)}(\vec{k})]$$

and use the anticommutation relations (11) along with the nonrelativistic approximation  $E = \sqrt{m^2 + \vec{p}^2} \approx m$ . As a result we obtain (in the vector meson rest frame  $\vec{P} = 0$ )

$$\begin{aligned} \langle P(P') | J_{\mu}(0) | V(P, \epsilon) \rangle &= \sqrt{4M_P M_V} \int \frac{d\vec{p}}{(2\pi)^3} \varphi_P^* \left( \frac{m_Q}{M_P} \vec{P}' + \vec{p} \right) \varphi_V(\vec{p}) \times \\ &\times \sum_{m s s' \bar{s}} C_{s' \bar{s}}^{00} C_{s \bar{s}}^{1m} \epsilon \cdot \epsilon_{(m)}^* \bar{u}_{(q)}^{(s')}(\vec{P}' + \vec{p}) \gamma_{\mu} (1 - \gamma_5) u_{(Q)}^{(s)}(\vec{p}). \end{aligned} \quad (12)$$

To simplify (12), note that

$$u^{(s)}(\vec{p}) = \frac{(\hat{p} + m)}{\sqrt{2m(p_0 + m)}} \chi^{(s)} \approx \frac{(\hat{p} + m)}{2m} \chi^{(s)}, \quad p_0 \approx m,$$

where  $\chi^{(s)}$  is the rest frame spinor, and

$$\begin{aligned} \sum_{m s s' \bar{s}} C_{s' \bar{s}}^{00} \epsilon \cdot \epsilon_{(m)}^* C_{s \bar{s}}^{1m} \chi^{(s)} \bar{\chi}^{(s')} &= \frac{1}{2} \{ (\chi^{(+)} \bar{\chi}^{(+)} - \chi^{(-)} \bar{\chi}^{(-)}) \epsilon \cdot \epsilon_{(0)}^* - \\ &- \sqrt{2} \chi^{(+)} \bar{\chi}^{(-)} \epsilon \cdot \epsilon_{(+)}^* + \sqrt{2} \chi^{(-)} \bar{\chi}^{(+)} \epsilon \cdot \epsilon_{(-)}^* \} = \\ &= \frac{1}{4} (1 + \gamma_0) \{ \gamma_3 \epsilon \cdot \epsilon_{(0)}^* + \gamma_+ \epsilon \cdot \epsilon_{(+)}^* + \gamma_- \epsilon \cdot \epsilon_{(-)}^* \} \gamma_5 = \frac{1}{4} (1 + \gamma_0) \vec{\gamma} \cdot \vec{\epsilon} \gamma_5. \end{aligned}$$

Here  $\gamma_+ = -\frac{1}{\sqrt{2}}(\gamma_1 + i\gamma_2)$ ,  $\gamma_- = \frac{1}{\sqrt{2}}(\gamma_1 - i\gamma_2)$  and  $\epsilon_{\pm}^* = -\epsilon_{\mp}$  property was used in the last step (note that  $\vec{\gamma} \cdot \vec{\epsilon} = \sum_{s=0, \pm} (-1)^s \gamma_{-s} \epsilon_{(s)}$ ).

Thus (12) transforms into

$$\begin{aligned} \langle P(P') | J_{\mu}(0) | V(P, \epsilon) \rangle &= \frac{\sqrt{M_P M_V}}{8m_q m_Q} \int \frac{d\vec{p}}{(2\pi)^3} \varphi_P^* \left( \frac{m_Q}{M_P} \vec{P}' + \vec{p} \right) \varphi_V(\vec{p}) \times \\ &\times Sp \{ (1 + \gamma_0) \vec{\gamma} \cdot \vec{\epsilon} \gamma_5 (\hat{p}' + m_q) \gamma_{\mu} (1 - \gamma_5) (\hat{p} + m_Q) \}, \end{aligned} \quad (13)$$

where  $p'_0 = m_q$  and  $\vec{p}' = \vec{P}' + \vec{p}$ .

It is now straightforward to extract the Lorentz-invariant form-factors from (13) once  $\varphi_P$  and  $\varphi_V$  wave functions are specified. It is assumed in

the ISGW approach that in the role of these wave functions one should use Schrödinger wave functions for the usual Coulomb plus linear potential that proved to be useful in quarkonia spectroscopy. But to simplify numerical calculations, they in fact used variational solutions of this Schrödinger problem based on Gaussian type harmonic-oscillator wave functions. In our case the relevant trial function is  $\varphi(\vec{r}) = \frac{\beta^{3/2}}{\pi^{3/4}} \exp(-\beta^2 r^2/2)$ , its momentum space image being

$$\varphi(\vec{p}) = \left( \frac{2\sqrt{\pi}}{\beta} \right)^{3/2} \exp(-\vec{p}^2/2\beta^2) \quad (14)$$

with  $\beta$  as variational parameter.

Let us introduce designations

$$\langle A_0 \rangle = \frac{1}{4} Sp \{ (1 + \gamma_0) \vec{\gamma} \cdot \vec{\epsilon} \gamma_5 (\hat{p}' + m_q) \gamma_0 \gamma_5 (\hat{p} + m_Q) \}$$

and analogously for  $\langle \vec{A} \rangle$ ,  $\langle V_0 \rangle$  and  $\langle \vec{V} \rangle$ . When we will have in the nonrelativistic limit [11]

$$\begin{aligned} \langle A_0 \rangle &= (m_Q + p_0) \vec{p}' \cdot \vec{\epsilon} + (m_q + p'_0) \vec{p} \cdot \vec{\epsilon} \rightarrow \\ &\rightarrow 2 \{ m_Q (\vec{P}' + \vec{p}) \cdot \vec{\epsilon} + m_q \vec{p} \cdot \vec{\epsilon} \}, \\ \langle \vec{A} \rangle &= \vec{\epsilon} \cdot \vec{p}' \vec{p} + \vec{\epsilon} \cdot \vec{p} \vec{p}' + p' \cdot p \vec{\epsilon} + m_q p_0 \vec{\epsilon} + m_Q p'_0 \vec{\epsilon} + m_q m_Q \vec{\epsilon} \rightarrow \\ &\rightarrow 4m_q m_Q \vec{\epsilon} - \vec{p} \cdot (\vec{P}' + \vec{p}) \vec{\epsilon} + (\vec{p} \cdot \vec{\epsilon}) (\vec{P}' + \vec{p}) + (\vec{P}' \cdot \vec{\epsilon}) \vec{p} + (\vec{p} \cdot \vec{\epsilon}) \vec{p}, \quad (15) \\ \langle \vec{V} \rangle &= i \{ (m_Q + p_0) \vec{\epsilon} \times \vec{p}' - (m_q + p'_0) \vec{\epsilon} \times \vec{p} \} \rightarrow \\ &\rightarrow 2i \{ m_Q \vec{\epsilon} \times (\vec{P}' + \vec{p}) - m_q \vec{\epsilon} \times \vec{p} \}. \end{aligned}$$

Using the last expression in (15) along with the equalities

$$\begin{aligned} \int \frac{d\vec{p}}{(2\pi)^3} \varphi_P^* \left( \frac{m_Q}{M_P} \vec{P}' + \vec{p} \right) \varphi_V(\vec{p}) &= \\ &= \left( \frac{\beta_P \beta_V}{\beta_{PV}^2} \right)^{3/2} \exp \left\{ -\frac{m_Q^2}{4M_P M_V} \frac{t_m - t}{\beta_{PV}^2} \right\} \equiv F(t), \end{aligned} \quad (16)$$

where  $\beta_{PV}^2 = \frac{1}{2}(\beta_P^2 + \beta_V^2)$ , and

$$\int \frac{d\vec{p}}{(2\pi)^3} \varphi_P^* \left( \frac{m_Q}{M_P} \vec{P}' + \vec{p} \right) \vec{p} \varphi_V(\vec{p}) = -\frac{m_Q}{M_P} \frac{\beta_V^2}{2\beta_{PV}^2} F(t) \vec{P}', \quad (17)$$

we get from (13) (it is supposed that the vector weak current  $\vec{V}(0)$  will be not confused with the vector meson  $V$ )

$$\begin{aligned} \langle P(P') | \vec{V}(0) | V(P, \epsilon) \rangle &= \\ &= i\vec{\epsilon} \times \vec{P}' \sqrt{M_P M_V} \left\{ \frac{1}{m_q} - \frac{1}{2\mu_-} \frac{m_Q}{M_P} \frac{\beta_V^2}{\beta_{PV}^2} \right\} F(t), \end{aligned}$$

where

$$\mu_{\pm} = \left[ \frac{1}{m_q} \pm \frac{1}{m_Q} \right]^{-1}. \quad (18)$$

On the other hand according to (16) we should have in the  $\vec{P} = 0$  frame

$$\langle P | \vec{V}(0) | V \rangle = 2iM_V g \vec{\epsilon} \times \vec{P}'.$$

Comparing these two expressions, we immediately get

$$g = \frac{1}{2} \sqrt{\frac{M_P}{M_V}} F(t) \left\{ \frac{1}{m_q} - \frac{1}{2\mu_-} \frac{m_Q}{M_P} \frac{\beta_V^2}{\beta_{PV}^2} \right\}. \quad (19)$$

Analogously the first equation in (15) leads to

$$\begin{aligned} a_+(M_P + M_V) + a_-(M_V - M_P) &= \\ &= -\sqrt{M_P M_V} \left\{ \frac{1}{m_q} - \frac{1}{2\mu_+} \frac{m_Q}{M_P} \frac{\beta_V^2}{\beta_{PV}^2} \right\} F(t). \end{aligned} \quad (20)$$

There is some subtlety in using the equation for  $\langle \vec{A} \rangle$  from (15). For  $\vec{\epsilon} \perp \vec{P}'$  polarization it readily gives

$$f = 2\sqrt{M_P M_V} F(t), \quad (21)$$

while for  $\vec{\epsilon} \parallel \vec{P}'$  polarization it involves  $\sim \vec{p}^2$  terms about which there is no guarantee in our nonrelativistic approach. Nevertheless one can get the correct answer by separating  $D$ -wave partial amplitude, because there is nothing intrinsically relativistic in recoiling into a  $D$  wave [9]. So let us disregard  $\sim \vec{\epsilon}$  terms from  $\langle \vec{A} \rangle$ , that correspond to a  $S$ -wave, and also in

$$\int \frac{d\vec{p}}{(2\pi)^3} \varphi_P^*(\vec{p} + \vec{q}) p_i p_j \varphi_V(\vec{p}) = A \vec{q}^2 \delta_{ij} + B q_i q_j, \quad (22)$$

omit the first term, which leads to  $S$ -wave amplitude too. Using

$$\int \frac{d\vec{p}}{(2\pi)^3} \varphi_P^*(\vec{p} + \vec{q}) \vec{p}^2 \varphi_V(\vec{p}) = \left[ \frac{3}{2} \frac{\beta_P^2 \beta_V^2}{\beta_{PV}^2} + \frac{q^2}{4} \frac{\beta_V^4}{\beta_{PV}^4} \right] F(t)$$

and

$$\int \frac{d\vec{p}}{(2\pi)^3} \varphi_P^*(\vec{p} + \vec{q}) (\vec{p} \cdot \vec{q})^2 \varphi_V(\vec{p}) = \left[ \frac{1}{2} \frac{\beta_P^2 \beta_V^2}{\beta_{PV}^2} + \frac{q^2}{4} \frac{\beta_V^4}{\beta_{PV}^4} \right] \vec{q}^2 F(t),$$

we easily obtain

$$B = \frac{1}{4} \frac{\beta_V^4}{\beta_{PV}^4} F(t). \quad (23)$$

Now we have all necessary ingredients to get a relation which follows from the  $D$ -wave relevant terms of  $\langle \vec{A} \rangle$ :

$$a_+ - a_- = \frac{\sqrt{M_P M_V}}{m_q m_Q} F(t) \left[ \frac{m_Q}{M_P} \frac{\beta_V^2}{2\beta_{PV}^2} - \frac{1}{4} \frac{m_Q^2}{M_P^2} \frac{\beta_V^4}{\beta_{PV}^4} \right]. \quad (24)$$

From (20) and (24)  $a_+$  form factor can be evaluated. Nothing that in the weak binding approximation  $\frac{M_V - M_P}{m_q m_Q} \approx \frac{m_Q - m_q}{m_q m_Q} = \frac{1}{\mu_-}$ , we get

$$a_+ = \sqrt{\frac{M_P}{M_V}} F(t) \left\{ \frac{1}{2m_q} \frac{m_Q}{M_P} \frac{\beta_V^2}{\beta_{PV}^2} - \frac{1}{8\mu_-} \frac{m_Q^2}{M_P^2} \frac{\beta_V^4}{\beta_{PV}^4} - \frac{1}{2m_q} \right\}.$$

Let us further transform

$$\begin{aligned} \frac{1}{m_q} \frac{m_Q}{M_P} \frac{\beta_V^2}{\beta_{PV}^2} &= \frac{1}{m_q} \frac{m_Q}{M_P} \frac{(\beta_V^2 - \beta_P^2) + (\beta_V^2 + \beta_P^2)}{(\beta_V^2 + \beta_P^2)} = \\ &= \frac{1}{m_q} \frac{m_Q}{M_P} \frac{\beta_V^2 - \beta_P^2}{\beta_V^2 + \beta_P^2} + \frac{1}{m_q} \frac{m_Q + m_q - m_q}{M_P} \approx \\ &\approx \frac{1}{m_q} - \frac{1}{M_P} + \frac{1}{M_P} \frac{m_Q}{m_q} \frac{\beta_V^2 - \beta_P^2}{\beta_V^2 + \beta_P^2}. \end{aligned}$$

This enables to rewrite  $a_+$  form factor as

$$a_+ = \sqrt{\frac{M_P}{M_V}} \frac{F(t)}{2M_P} \left[ -1 + \frac{m_Q}{m_q} \frac{\beta_V^2 - \beta_P^2}{\beta_V^2 + \beta_P^2} - \frac{1}{4\mu_-} \frac{m_Q^2}{M_P} \frac{\beta_V^4}{\beta_{PV}^4} \right]. \quad (25)$$

Having at hand (19), (21) and (25) expressions for the  $g$ ,  $f$  and  $a_+$  form factors, the semileptonic decay width can be evaluated through (4), (5) and (7) formulas.

## 4 BSW model

The Bauer-Stech-Wirbel model [8,12] uses the quark model to deal only with one point  $q^2 = 0$ . In contrast to the zero recoil point, considered previously in the ISGW model,  $q^2 = 0$  point can be highly relativistic. So the relativistic treatment of quark dynamics becomes unavoidable, although this dynamics greatly simplifies in the Infinite Momentum Frame. It is convenient to represent meson state vectors in this frame in the slightly different from (10) form

$$\begin{aligned}
 |V(P, \epsilon)\rangle &= \sqrt{2} \sum_{s\bar{s}m} \int \frac{d\vec{p}_1 d\vec{p}_2}{(2\pi)^{3/2}} \sqrt{\frac{m_Q m_q}{p_{10} p_{20}}} \delta(\vec{P} - \vec{p}_1 - \vec{p}_2) \times \\
 &\times C_{s\bar{s}}^{1m} \epsilon \cdot \epsilon_m^* \varphi_V(\vec{p}_1) |Q[\vec{p}_1, s] \bar{Q}[\vec{p}_2, \bar{s}] \rangle, \\
 |P(P')\rangle &= \sqrt{2} \sum_{s\bar{s}m} \int \frac{d\vec{p}_1 d\vec{p}_2}{(2\pi)^{3/2}} \sqrt{\frac{m_q m_Q}{p_{10} p_{20}}} \delta(\vec{P}' - \vec{p}_1 - \vec{p}_2) \times \\
 &\times C_{s\bar{s}}^{00} \varphi_P(\vec{p}_1) |q[\vec{p}_1, s] \bar{Q}[\vec{p}_2, \bar{s}] \rangle.
 \end{aligned} \quad (26)$$

In the Infinite Momentum Frame and for  $\vec{q} = \vec{P}_V - \vec{P}_P = 0$  we have  $P_{V\mu} = (E_V, 0, 0, P)$ ,  $P_{P\mu} = (E_P, 0, 0, P)$ ,  $P \rightarrow \infty$ . But  $E_V - E_P \approx \frac{M_V^2 - M_P^2}{2P} \rightarrow 0$ . That is  $\vec{q} = 0$  just gives  $q^2 = 0$  point.

Let us introduce the longitudinal momentum fraction carried by the active quark in the meson  $x = \frac{p_{1+}}{P}$ , when the normalization condition for the  $\varphi(\vec{p}_1)$  wave function, which follows from (26), is

$$\int dx d\vec{p}_T |\varphi(x, \vec{p}_T)|^2 = 1. \quad (27)$$

The concrete form of this wave function is inspired by the relativistic harmonic oscillator model and looks like [8] (for the meson of mass  $M$  made up from active quark  $q$  and spectator antiquark  $\bar{Q}$ )

$$\begin{aligned}
 \varphi(x, \vec{p}_T) &= N \sqrt{x(1-x)} \exp\left(-\frac{\vec{p}_T^2}{2\omega^2}\right) \times \\
 &\times \exp\left\{-\frac{M^2}{2\omega^2} \left(x - \frac{1}{2} - \frac{m_q^2 - m_Q^2}{2M^2}\right)^2\right\},
 \end{aligned} \quad (28)$$

where  $N$  is determined from the normalization condition (27). The dimensional parameter  $\omega$  controls transverse momentum suppression and equals to

the average transverse momentum  $\omega^2 = \langle \vec{p}_T^2 \rangle$ . In the role of  $\omega$  we can use  $\beta$  parameter from (14), as  $\vec{p}_T$  is not changed by the boost along  $z$ -direction.

Manipulations which had led to (12), now give for  $\vec{q} = \vec{P}_V - \vec{P}_P = 0$

$$\begin{aligned}
 \langle P(P) | J_\mu(0) | V(P, \epsilon) \rangle &= 2 \sum_{m s s' \bar{s}} \int d\vec{p} \sqrt{\frac{m_q m_Q}{p_0 p'_0}} C_{s' \bar{s}}^{00} C_{s \bar{s}}^{1m} \times \\
 &\times \epsilon \cdot \epsilon_m^* \varphi_P^*(\vec{p}) \varphi_V(\vec{p}) \bar{u}_{(q)}^{(s')}(\vec{p}) \gamma_\mu (1 - \gamma_5) u_{(Q)}^{(s)}(\vec{p}).
 \end{aligned} \quad (29)$$

In the infinite momentum limit  $p_0 = \sqrt{m_Q^2 + \vec{p}^2} = \sqrt{m_Q^2 + x^2 P^2 + \vec{p}_T^2} \rightarrow xP$ ,  $p'_0 = \sqrt{m_q^2 + \vec{p}^2} \rightarrow xP$  and

$$u^{(s)}(\vec{p}) = \frac{\hat{p} + m}{\sqrt{2m(p_0 + m)}} \chi^{(s)} \rightarrow \frac{\hat{p} + m}{\sqrt{2mxP}} \chi^{(s)}.$$

So (29) transforms into

$$\begin{aligned}
 \langle P | J_\mu(0) | V \rangle &= \int dx d\vec{p}_T \frac{\varphi_P^*(\vec{p}) \varphi_V(\vec{p})}{x^2 P} \times \\
 &\times S_P \left\{ \frac{1}{4} (1 + \gamma_0) \vec{\gamma} \cdot \vec{\epsilon} \gamma_5 (\hat{p}' + m_q) \gamma_\mu (1 - \gamma_5) (\hat{p} + m_Q) \right\},
 \end{aligned} \quad (30)$$

where  $\vec{p}' = \vec{p}$  and  $p'_0 \rightarrow xP$ .

From (15) we will have in the  $P \rightarrow \infty$  limit

$$\langle \vec{V} \rangle \rightarrow i(m_Q - m_q) \vec{\epsilon} \times \vec{p}.$$

On the other hand according to (6)

$$\langle P | \vec{V}(0) | V \rangle = 2ig(E_V - E_P) \vec{\epsilon} \times \vec{P}_V \rightarrow ig \frac{M_V^2 - M_P^2}{P} \vec{\epsilon} \times \vec{P}_V.$$

Comparing these expressions of  $\langle P | \vec{V}(0) | V \rangle$  and using

$$\int d\vec{p} \frac{1}{x^2 P} \varphi_P^*(\vec{p}) \vec{p} \varphi_V(\vec{p}) \rightarrow \frac{1}{P} J \vec{\epsilon} \times \vec{P}_V,$$

where

$$J = \int d\vec{p}_T \int_0^1 \frac{dx}{x} \varphi_P^*(x, \vec{p}_T) \varphi_V(x, \vec{p}_T), \quad (31)$$

we get

$$g(q^2 = 0) = \frac{m_Q - m_q}{M_V^2 - M_P^2} J. \quad (32)$$

But this gives the form factor only at one  $q^2 = 0$  point. For values of  $q^2$  others than zero, the BSW model assumes nearest pole dominance:

$$A_1(q^2) = \frac{h_{A_1}}{1 - \frac{q^2}{M_{1^+}^2}}, \quad A_2(q^2) = \frac{h_{A_2}}{1 - \frac{q^2}{M_{1^+}^2}}, \quad V_1(q^2) = \frac{h_V}{1 - \frac{q^2}{M_{1^-}^2}}, \quad (33)$$

Thus the  $q^2$ -dependence of form factors are determined once the masses of the appropriate  $1^-$  and  $1^+$  vector mesons are known.

Then (32) indicates, that

$$h_V = \frac{m_Q - m_q}{M_V - M_P} J. \quad (34)$$

Analogously, using  $\langle \vec{A} \rangle \rightarrow x(m_Q + m_q)P \vec{\epsilon} + 2(\vec{\epsilon} \cdot \vec{p}) \vec{p}$ ,  $\langle P | \vec{A}(0) | V \rangle = f \vec{\epsilon} + 2(\epsilon \cdot P_P) a_+ \vec{P}_V$  and  $\epsilon \cdot P_P \rightarrow (\frac{E_P}{E_V} - 1) \vec{\epsilon} \cdot \vec{P}_V$ , we can get  $f = (M_Q + M_q)J$ , and so

$$h_{A_1} = \frac{m_Q + m_q}{M_V + M_P} J. \quad (35)$$

Again there is a subtlety in extracting  $a_+$ . Instead of giving a rigorous derivation, we prefer the following educative guess. Noting that for  $P_{V\mu} = (E_V, 0, 0, P)$  the longitudinal polarization 4-vector  $\epsilon_{\parallel\mu} = \frac{1}{M_V}(P, 0, 0, E_V) \rightarrow \frac{P}{M_V}(1, 0, 0, 1)$ , we obtain

$$\begin{aligned} \int d\vec{x} \langle P | A_0(x) | V \rangle &= (2\pi)^3 \delta(\vec{P}_V - \vec{P}_P) \langle P | A_0(0) | V \rangle \rightarrow \\ &\rightarrow (2\pi)^3 \delta(\vec{P}_V - \vec{P}_P) \left\{ f \epsilon_0 + \left( \frac{E_P}{E_V} - 1 \right) \epsilon_3 P (E_V + E_P) a_+ \right\} \rightarrow \\ &\rightarrow (2\pi)^3 \delta(\vec{P}_V - \vec{P}_P) \left\{ f - (M_V^2 - M_P^2) a_+ \right\} \frac{P}{M_V}. \end{aligned}$$

On the other hand  $Q_{50} = \int d\vec{x} A_0(x)$  is an appropriate weak charge, which in the exact flavor symmetry limit transforms  $|V\rangle$  initial state into  $|P\rangle$  final state and so

$$\int d\vec{x} \langle P | A_0(x) | V \rangle = \langle P(P_P) | Q_{50} | V(P_V) \rangle =$$

$$= \langle P(P_P) | P(P_V) \rangle \rightarrow 2P(2\pi)^3 \delta(\vec{P}_V - \vec{P}_P).$$

In the broken flavor symmetry case one should expect instead

$$\langle P(P_P) | Q_{50} | V(P_V) \rangle = 2PI(2\pi)^3 \delta(\vec{P}_V - \vec{P}_P),$$

with  $I$  as the wave function overlap integral.

$$I = \int d\vec{p}_T \int_0^1 dx \varphi_P^*(x, \vec{p}_T) \varphi_V(x, \vec{p}_T). \quad (36)$$

Thus we obtain

$$a_+ = \frac{1}{M_V^2 - M_P^2} [f - 2M_V I]$$

and so

$$h_{A_2} = \frac{2M_V}{M_V - M_P} I - \frac{M_V + M_P}{M_V - M_P} h_{A_1}. \quad (37)$$

(34), (35) and (36) formulas and the nearest pole dominance hypothesis completely determine the weak form factors in the BSW model.

## 5 Heavy quark limit

In the limit in which the quarks active in weak transition are very heavy, all form factors for this transition can be expressed in terms of a single function  $\xi(\zeta)$  called Isgur-Wise function [13]. In the case of  $1^- \rightarrow 0^-$  transitions these relations look like

$$\begin{aligned} A_1 &= \frac{\sqrt{M_P M_V}}{M_P + M_V} (1 + \zeta) \xi(\zeta), \\ A_2 = V &= \frac{1}{2} \sqrt{\frac{M_P}{M_V}} \left( 1 + \frac{M_V}{M_P} \right) \xi(\zeta), \end{aligned} \quad (38)$$

where

$$\zeta = v_P \cdot v_V = \frac{M_P^2 + M_V^2 - q^2}{2M_V M_P}.$$

Again some dynamical model of mesons is needed to calculate the Isgur-Wise function  $\xi(\zeta)$  (as an example of such calculations see [14, 15]). But one can use instead some phenomenologically successful parameterization. In particular, the following parameterizations was shown [16] to fit experimental



data reasonably well

$$\begin{aligned}\xi(\zeta) &= 1 - \rho^2(\zeta - 1), \quad \rho \approx 1.08; \\ \xi(\zeta) &= \frac{2}{1+\zeta} \exp\left\{-(2\rho^2 - 1)\frac{\zeta-1}{\zeta+1}\right\}, \quad \rho \approx 1.52; \\ \xi(\zeta) &= \left(\frac{2}{\zeta+1}\right)^{2\rho^2}, \quad \rho \approx 1.45; \\ \xi(\zeta) &= \exp\{-\rho^2(\zeta - 1)\}, \quad \rho \approx 1.37.\end{aligned}\quad (39)$$

In our case, heavy quark limit can be applied to  $\Upsilon \rightarrow B_c^+ e^- \bar{\nu}_e$  decay. Despite of different analytical forms of the Isgur-Wise function, all four parameterizations from (39) give essentially the same  $Br(\Upsilon \rightarrow B_c^+ e^- \bar{\nu}_e)$ :  $4.1 \cdot 10^{-10}$ ,  $3.7 \cdot 10^{-10}$ ,  $3.8 \cdot 10^{-10}$  and  $3.8 \cdot 10^{-10}$  respectively.

For heavy-light transitions, as for example in  $J/\psi \rightarrow D_d^- e^+ \nu_e$  decay, the Isgur-Wise scaling (38) is not applicable. Recently B. Stech proposed [17] a phenomenological model for semileptonic form factors which generalizes the Isgur-Wise scaling. It is supposed that instead of (38) the following relations hold

$$\begin{aligned}A_1 &= \frac{\sqrt{M_P M_V}}{M_P + M_V} (1 + \zeta) h_{A_1}(\zeta) \xi_{PV}(\zeta), \quad A_2 = \frac{1}{2} \sqrt{\frac{M_P}{M_V}} \left(1 + \frac{M_V}{M_P}\right) h_{A_2}(\zeta) \xi_{PV}(\zeta), \\ V &= \frac{1}{2} \sqrt{\frac{M_P}{M_V}} \left(1 + \frac{M_V}{M_P}\right) h_V(\zeta) \xi_{PV}(\zeta).\end{aligned}\quad (40)$$

The function  $\xi_{PV}(\zeta)$  is the same for all form factors for the given initial and final states. It approaches the Isgur-Wise function in the heavy quark limit. On contrary,  $h$ -functions are different for each form factor and approach unit in the heavy quark limit. The concrete expressions for the  $\xi_{PV}(\zeta)$  and  $h$ -functions can be found in the original paper [17].

## 6 Numerical results

To perform numerical calculations within the ISGW model framework, one needs to specify quark masses and  $\beta$  variational parameters. We use the following values for quark masses [10]

$$m_u = m_d = 0.33 \text{ GeV}, \quad m_s = 0.55 \text{ GeV}, \quad m_c = 1.82 \text{ GeV}, \quad m_b = 5.12 \text{ GeV},$$

and  $\beta$  parameters (in GeV)

$$\begin{aligned}\beta_k &= 0.44, \quad \beta_{D_d} = 0.45, \quad \beta_{D_s} = 0.56, \quad \beta_{B_u} = 0.43, \\ \beta_{B_c} &= 0.92, \quad \beta_\varphi = 0.37, \quad \beta_{J/\psi} = 0.62, \quad \beta_\Upsilon = 1.1.\end{aligned}\quad (41)$$

All but the last values in (41) are from Table A2 of ref.[10]. The value for  $\Upsilon$  was obtained by minimizing  $\langle \frac{\vec{p}^2}{m_b} + V \rangle$ , with (14) as a trial function and  $V(r) = -\frac{4\alpha_s}{3r} + C + br$ , where [10]  $\alpha_s \approx 0.3$ ,  $b = 0.18 \text{ GeV}^2$ ,  $C = -0.84 \text{ GeV}$ . This minimization problem leads to a cubic equation

$$\beta^3 - \frac{8\alpha_s m_b}{9\sqrt{\pi}} \beta^2 - \frac{2bm_b}{3\sqrt{\pi}} = 0$$

with  $\beta \approx 1.1$  as a solution.

Note that this variational solution corresponds to the  $\Upsilon$ -meson mass

$$M_\Upsilon = 2m_b + \langle \frac{\vec{p}^2}{m_b} + V \rangle \approx 9.44 \text{ GeV},$$

which should be compared to the experimental value [18] 9.46 GeV.

As was already mentioned, the ISGW model predictions about the high  $(t_m - t)$ -behavior of form factors are not reliable. In numerical calculations we use more realistic behavior, suggested in [10] (although we don't use other refinements of the model given in [10])

$$F(t) \rightarrow \left(\frac{\beta_P \beta_V}{\beta_{PV}^2}\right)^{3/2} \left[1 + \frac{1}{12} r^2 (t_m - t)\right]^{-2}, \quad (42)$$

with

$$r^2 = \frac{3}{4m_q m_Q} + \frac{3m_Q^2}{2M_P M_V \beta_{PV}^2} + \frac{\Delta r^2}{M_P M_V}. \quad (43)$$

The last term in (43) differs from zero only for  $b \rightarrow c$  transitions and equals [10]  $\Delta r^2 \approx 0.39$ .

For the BSW model one needs in addition  $1^-$  and  $1^+$  pole masses to define the form factors  $q^2$ -dependence. We use the following values beauty-charm mesons are not yet discovered experimentally. Predictions for their masses were taken from [19] (in particular,  $M_{B_c} = 6.253 \text{ GeV}$ ). Value of  $M_{1^+} = 5.745 \text{ GeV}$  for  $(b\bar{u})$ -meson is also a potential model prediction taken from [20].

Table 1:

Decay	$\varphi \rightarrow K^+ e^- \bar{\nu}$	$J/\psi \rightarrow D_d^- e^+ \nu$	$J/\psi \rightarrow D_s^- e^+ \nu$	$\Upsilon \rightarrow B_u^+ e^- \bar{\nu}$	$\Upsilon \rightarrow B_c^+ e^- \bar{\nu}$
$M_{1^+}, \text{ GeV}$	1.273( $K_1$ )	2.422( $D_1$ )	2.535( $D_{s1}$ )	5.745	6.717
$M_{1^-}, \text{ GeV}$	0.892( $K^*$ )	2.010( $D^*$ )	2.112( $D_s^*$ )	5.325( $B^*$ )	6.317

As was already mentioned earlier, we consider  $\omega$ -parameter of the BSW model in (28) to be the same as the corresponding  $\beta$ -parameter of the ISGW model from (14). For the  $\Upsilon \rightarrow B_c^- e^- \bar{\nu}$  decay this choice gives 5-times smaller branching than it is expected from the heavy quark limit. Especially sensitive to this parameter is  $\text{Br}(\Upsilon \rightarrow B_c^- e^- \bar{\nu})$ , which is in fact determined by the overlap of the wave function tails, and it is hard to expect that this tails are correctly given by the simple parameterization used in the BSW model. So we decided that it is more reasonable to choose  $\omega_\Upsilon$  such that the heavy quark limit prediction is reproduced, as much as it is possible, for the  $\text{Br}(\Upsilon \rightarrow B_c^- e^- \bar{\nu})$ . This gives  $\omega_\Upsilon \approx 2.2\text{GeV}$  as compared to  $\beta_\Upsilon \approx 1.1\text{GeV}$  of the ISGW model. For other quarkonia  $\omega = \beta$  prescription was used.

The numerical results for various semileptonic branching ratios are summarized in the table below.

Table 2:

Decay	$\varphi \rightarrow K^+ e^- \bar{\nu}$	$J/\psi \rightarrow D_s^- e^+ \nu$	$J/\psi \rightarrow D_s^- e^+ \nu$	$\Upsilon \rightarrow B_u^+ e^- \bar{\nu}$	$\Upsilon \rightarrow B_c^+ e^- \bar{\nu}$
ISGW [9]	$7.9 \cdot 10^{-15}$	$2.3 \cdot 10^{-11}$	$4.8 \cdot 10^{-10}$	$2.9 \cdot 10^{-13}$	$1.6 \cdot 10^{-10}$
BSW [8]	$3.1 \cdot 10^{-14}$	$3.9 \cdot 10^{-11}$	$8.9 \cdot 10^{-10}$	$3.5 \cdot 10^{-13}$	$2.0 \cdot 10^{-10}$
Stech [17]	-	$3.1 \cdot 10^{-11}$	$5.2 \cdot 10^{-10}$	$3.0 \cdot 10^{-12}$	$3.1 \cdot 10^{-10}$

## 7 Conclusions

We have considered some semileptonic weak decays of vector mesons, using the most popular ISGW and BSW quark models. The predictions of these models agree to each other reasonably well (within a factor 2), except  $\varphi \rightarrow K^+ e^- \bar{\nu}$  decay, where predicted branchings differ 4-times.

The corresponding branching ratios were also calculated using recently proposed Stech's phenomenological model [17]. The results agree again with the ISGW and BSW models predictions, except  $\varphi \rightarrow K^+ e^- \bar{\nu}$  and  $\Upsilon \rightarrow B_u^+ e^- \bar{\nu}$  decays. As for the  $\varphi \rightarrow K^+ e^- \bar{\nu}$  decay, for which the result is  $\text{Br}(\varphi \rightarrow K^+ e^- \bar{\nu}) = 2.7 \cdot 10^{-12}$ , we don't expect Stech's model to be valid for it. But it is interesting to note that if we don't require, as in [17],  $\xi_{PV}(\zeta)$  to have a pole in  $q^2$  at the position of the lowest  $0^-$  resonance (the pseudoscalar  $P$  meson itself), but instead demand that the pole position for  $\xi_{PV}(\zeta)$  depends on the form-factor, in which  $\xi_{PV}(\zeta)$  enters, exactly as in the BSW model (that is  $1^-$ -pole for the  $V$  form-factor and  $1^+$ -pole for the  $A_1$  and  $A_2$  form-factors), then so modified Stech's model predicts  $\text{Br}(\varphi \rightarrow K^+ e^- \bar{\nu}) = 9.0 \cdot 10^{-15}$ , again close

to the ISGW and BSW results. The other decay modes are not significantly effected by this modification. In particular, an order of magnitude difference between Stech's model on one hand and ISGW or BSW model on another for the  $\Upsilon \rightarrow B_u^+ e^- \bar{\nu}$  decay still persists. It seems to us that the Stech's model has difficulties in handling this decay mode.

Unfortunately, the predicted branching ratios are too small and so an experimental study of the decays considered is questionable in near future.

## Acknowledgments

We are grateful to Victor Chernyak for useful discussions.

## References

- [1] A. Le Yaouanc, Nucl. Instr. Meth. **A351**(1994), 15.
- [2] N. Isgur, Models of semileptonic decays, Invited talk given at 1989 Int. Conf. on Heavy Quark Physics, Cornell. Toronto University preprint UTPT-89-25, 1989.
- [3] M. Wirbel, Semileptonic B decays, Dortmund University preprint DO-TH-89/4, 1989
- [4] D. Melikhov, Phys. Rev. **D53**(1996), 2460.
- [5] G. Altarelli et al., Nucl. Phys. **B208**(1982), 365.  
N. Cabibbo, G. Corbo, L. Maiani, Nucl. Phys. **B155**(1979), 93.
- [6] B. Grinstein, M. B. Wise, N. Isgur, Phys. Rev. Lett. **56** (1986), 298.
- [7] D. Scora, N. Isgur, Phys. Rev. **D40**(1989), 1491.
- [8] M. Wirbel, B. Stech, M. Bauer, Z. Phys. **C29**(1985), 637.  
M. Bauer, M. Wirbel, Z. Phys. **C42**(1989), 671.
- [9] N. Isgur, D. Scora, B. Grinstein, M. B. Wise, Phys. Rev. **D39**(1989), 799.
- [10] D. Scora, N. Isgur, Phys. Rev. **D52**(1995), 2783.
- [11] T. Altomari, L. Wolfenstein, Phys. Rev. **D37**(1988), 681.

- [12] B. Konig, J. G. Korner, M. Kramer, P. Kroll, Infinite Momentum Frame calculation of semileptonic heavy  $\Lambda_b \rightarrow \Lambda_c$  transitions including HQET improvements, preprint DESY-93-011, 1993 (hep-ph/ 9701212).
- [13] N. Isgur, M. B. Wise, Phys. Lett. **B232**(1989), 113; **B237**(1990), 527. M. Neubert, V. Rieckert, Nucl. Phys. **B382**(1992), 97.
- [14] A. Le Yaouanc, L. Oliver, O. Pène, J. -C. Raynal Phys. Lett. **B365**(1996), 319.
- [15] D. Melikhov, Heavy quark expansion and universal form-factors in quark model, hep-ph/ 9706417.
- [16] H. Albrecht et al. (ARGUS coll.), Z. Phys. **C57**(1993), 533.
- [17] B. Stech, Z. Phys. **C75**(1997), 245; Nucl. Phys. Proc. Suppl. **50**(1996), 45.
- [18] Review of Particle Physics, Phys. Rev. **D54**(1996).
- [19] S. S. Gershtein, V. V. Kiselev, A. K. Likhoded, A. V. Tkabladze, Usp. Fiz. Nauk **165**(1995), 3.
- [20] S. N. Gupta, J. M. Johnson, Phys. Rev. **D51**(1995), 168.

*L.M. Kurdadze, Z.K. Silagadze*

**On Some rare weak decays  
of vector mesons**

Budker INP 97-101

Ответственный за выпуск А.М. Кудрявцев

Работа поступила 17.12. 1997 г.

Сдано в набор 21.12.1997 г.

Подписано в печать 21.12.1997 г.

Формат бумаги 60×90 1/16 Объем 1.0 печ.л., 0.9 уч.-изд.л.

Тираж 100 экз. Бесплатно. Заказ № 101

Обработано на IBM PC и отпечатано на  
роталпринте ИЯФ им. Г.И. Будкера СО РАН,  
Новосибирск, 630090, пр. академика Лаврентьева, 11.