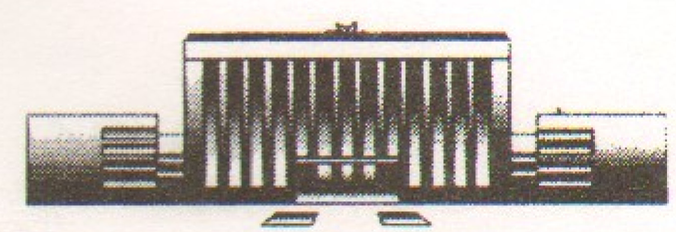
Budker Institute of Nuclear Physics
SB RAS

V.N. Baier and V.M. Katkov

THE LANDAU-POMERANCHUK-MIGDAL EFFECT IN A THIN TARGET

Budker INP 97-105



Novosibirsk

The Landau-Pomeranchuk-Migdal effect in a thin target

V.N. Baier and V.M. Katkov

Budker Institute of Nuclear Physics SB RAS
630090 Novosibirsk, Russia

Abstract

The Landau, Pomeranchuk, Migdal (LPM) effect (suppression of the bremsstrahlung from high energy electron due to a multiple scattering of an emitting electron in dense media) is considered for the case when thickness of a target is of the order or less than the formation length of radiation. The effects of the polarization of a medium and transition radiation are taken into account as well. Qualitative picture of the phenomenon is discussed in detail. Comparison with recent experimental data is carried out.

1 Introduction

The process of bremsstrahlung from high-energy electron occurs over a rather long distance, known as the formation length. If anything happens to an electron or a photon while traveling this distance, the emission can be disrupted. Landau and Pomeranchuk showed that if the formation length of bremsstrahlung becomes comparable to the distance over which a mean angle of multiple scattering becomes comparable with a characteristic angle of radiation, the bremsstrahlung will be suppressed [1]. Migdal [2], [3] developed a quantitative theory of this phenomenon. An influence of polarization of a medium on radiation process leads also to suppression of the soft photon emission (Ter-Mikaelian effect, see in [4]).

A very successful series of experiments [5] - [7] was performed at SLAC during last years. In these experiments the cross section of bremsstrahlung of soft photons with energy from 200 KeV to 500 MeV from electrons with energy 8 GeV and 25 GeV is measured with an accuracy of the order of a few percent. Both LPM and dielectric suppression is observed and investigated. These experiments were the challenge for the theory since in all the previous papers calculations (cited in [8]) are performed to logarithmic accuracy which is not enough for description of the new experiment. The contribution of the Coulomb corrections (at least for heavy elements) is larger then experimental errors and these corrections should be taken into account.

Very recently authors developed the new approach to the theory of LPM effect [8] where the cross section of bremsstrahlung process in the photon energies region where the influence of the LPN is very strong was calculated with term $\propto 1/L$, where L is characteristic logarithm of the problem, and with the Coulomb corrections taken into account. In the photon energy region, where the LPM effect is "turned off", the obtained cross section gives the exact Bethe-Heitler cross section (within power accuracy) with Coulomb corrections. This important feature was absent in the previous calculations. The contribution of an inelastic scattering of a projectile on atomic electrons is also included. The polarization of a medium is incorporated into this approach. The considerable contribution into the soft part of the measured spectrum of radiation gives a photon emission on the boundaries of a target. We calculated this contribution taking into account the multiple scattering and polarization of a medium for the case when a target is much thicker than the formation length of the radiation. We considered also a case when a target is much thinner than the formation length. A case of an intermediate thickness of a target (between cases of a thick and a thin target) is analyzed but polarization of a medium is not included.

In the present paper we calculated the cross section of bremsstrahlung process in a target of intermediate thickness. In Section 2 we derived general expression for the spectral probability of radiation in a thin target and in a target with intermediate thickness where the multiple scattering, the polarization of a medium and radiation on boundaries of a target are taken into account. The representations suitable for numerical calculations are derived. Useful asymptotic formulae are found. In Section 3 qualitative picture of the phenomenon is discussed in detail. In Section 4 we compare the calculated spectral curves with recent experimental data [7] where electrons with energy $\varepsilon = 25\ GeV$ and $\varepsilon = 8\ GeV$ radiated in a gold target with thickness $l = 0.7(0.1)\%\ L_{rad}$. Agreement between theory and data is perfect for $l = 0.7\%\ L_{rad}$ at electron energy $\varepsilon = 25\ GeV$, for the same target and $\varepsilon = 8\ GeV$ agreement is satisfactory.

2 Spectral distribution of the probability of radiation

Proceeding from the formulation of [8] (see Section 4) we can obtain general expression which takes into account boundary effects for a target of arbitrary thickness. With allowance for multiple scattering and polarization of a medium we have for the spectral distribution of the probability of radiation

$$\frac{dw}{d\omega} = \frac{4\alpha}{\omega}\mathrm{Re}\int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{t_2} dt_1 \exp\left(-i\int_{t_1}^{t_2}\mu(t)dt\right) \times \langle 0|r_1 S(t_2,t_1) + r_2 \mathbf{p} S(t_2,t_1)\mathbf{p}|0\rangle, \tag{2.1}$$

where

$$\mu(t) = \vartheta(-t) + \vartheta(t-T) + \kappa\vartheta(t)\vartheta(T-t),\quad T = \frac{l}{l_0},\quad l_0 = \frac{2\varepsilon\varepsilon'}{\omega m^2},$$
$$r_1 = \frac{\omega^2}{\varepsilon^2},\quad r_2 = 1 + \frac{\varepsilon'^2}{\varepsilon^2},\quad \kappa = 1 + \kappa_0^2,\quad \kappa_0 = \frac{\omega_p}{\omega}, \tag{2.2}$$
$$\omega_p = \gamma\omega_0,\quad \gamma = \frac{\varepsilon}{m},\quad \omega_0^2 = \frac{4\pi\alpha n}{m},$$

here ε is the energy of the initial electron, ω is the energy of radiated photon, $\varepsilon' = \varepsilon - \omega$, n is the density of the atoms in a medium, l is the thickness of a target. So, we split time interval (in the used units) into three parts: before target ($t < 0$), after target ($t > T$) and inside target ($0 \leq t \leq T$). The mean value in Eq.(2.1) is taken over states with definite value of the two-dimensional operator ϱ (see [8], Section 2). The propagator of electron has a form

$$S(t_2,t_1) = \mathrm{T}\exp\left[-i\int_{t_1}^{t_2}\mathcal{H}(t)dt\right], \tag{2.3}$$

where the Hamiltonian $\mathcal{H}(t)$ in the case of a homogeneous medium is

$$\mathcal{H}(t) = \mathbf{p}^2 - iV(\varrho)g(t),\quad \mathbf{p} = -i\nabla_{\varrho},\quad g(t) = \vartheta(t)\vartheta(T-t),$$
$$V(\varrho) = Q\varrho^2\left(L_1 + \ln\frac{4}{\varrho^2} - 2C\right),\quad Q = \frac{2\pi Z^2\alpha^2\varepsilon\varepsilon' n}{m^4\omega},\quad L_1 = \ln\frac{a_{s2}^2}{\lambda_c^2},$$
$$\frac{a_{s2}}{\lambda_c} = 183 Z^{-1/3}e^{-f},\quad f = f(Z\alpha) = (Z\alpha)^2\sum_{k=1}^{\infty}\frac{1}{k(k^2+(Z\alpha)^2)},\quad C = 0.577.... \tag{2.4}$$

The contribution of scattering of a projectile on atomic electrons may be incorporated into effective potential $V(\varrho)$. The summary potential including both an elastic and an inelastic scattering is

$$V(\varrho) + V_e(\varrho) = -Q_{ef}\varrho^2\left(\ln\frac{\lambda_c^2}{a_{ef}^2} + \ln\frac{\varrho^2}{4} + 2C\right), \tag{2.5}$$

where

$$Q_{ef} = Q\left(1 + \frac{1}{Z}\right),\quad a_{ef} = a_{s2}\exp\left[\frac{1.88 + f(Z\alpha)}{1+Z}\right].$$

In (2.1) it is implied that subtraction is made at $V = 0$, $\kappa = 1$.

It is important to note that Eq.(4.1) of Ref.8 is valid for description of effects of multiple scattering and polarization of a medium. But for description of the transition radiation on two boundaries it should be modified as it is done in Eq.(2.1).

In [8] the potential $V(\varrho)$ was presented in the form

$$V(\varrho) = V_c(\varrho) + v(\varrho), \quad V_c(\varrho) = q\varrho^2, \quad q = QL,$$
$$L \equiv L(\varrho_c) = \ln\frac{a_{s2}^2}{\lambda_c^2\varrho_c^2}, \quad v(\varrho) = -\frac{q\varrho^2}{L}\left(\ln\frac{\varrho^2}{4\varrho_c^2} + 2C\right), \tag{2.6}$$

where the parameter ϱ_c is defined by a set of equations:

$$\varrho_c = 1 \text{ for } \nu_1 \le 1; \quad 4Q\varrho_c^4 L(\varrho_c) = 1 \text{ for } \nu_1 \ge 1; \quad \nu_1^2 \equiv 4QL_1. \tag{2.7}$$

This form is convenient for expansion over powers of $1/L$ (typical value $L \approx 10$).

The formation length of radiation (for $\omega \ll \varepsilon$) inside target with regard for the multiple scattering and the polarization of a medium (see Eqs.(3.3), (7.1) in [8])

$$l_f = \frac{2\gamma^2}{\omega}\left[1 + \gamma^2\vartheta_c^2 + \left(\frac{\gamma\omega_0}{\omega}\right)^2\right]^{-1} \tag{2.8}$$

can be written as

$$l_f = \frac{l_0}{\nu_0 + \kappa}, \quad \frac{l}{l_f} = T(\nu_0 + \kappa), \quad T = \frac{lm^2\omega}{2\varepsilon\varepsilon'};$$
$$\nu_0^2(\varrho_c) = 4QL(\varrho_c), \quad \nu_0^2(1) = \nu_1^2 \tag{2.9}$$

We calculated in [8] the probability of radiation inside a thick target taking into account the correction term $v(\varrho)$ to the potential $V_c(\varrho)$, see 2.6. This was important for sewing together with Bethe-Heitler cross section in the region of photon energies where influence of the multiple scattering is very weak ($\nu_1 \ll 1$). The contribution of boundary photons was calculated without the correction term $v(\varrho)$.

In the case when a target has intermediate thickness ($l \sim l_f$) the mentioned separation of contributions becomes senseless. We consider this case neglecting by the correction term $v(\varrho)$. The typical mean value we have to calculate (see (2.1)-(2.6)) is

$$\langle 0\left|\exp(iH_0t_1)\exp(-iHt_2)\right|0\rangle \to \langle 0\left|\exp(iH_0t_1)\exp(-iH_ct_2)\right|0\rangle =$$
$$\langle 0\left|\exp(iH_0t_1)\right|\varrho\rangle\langle\varrho\left|\exp(-iH_ct_2)\right|0\rangle = \int d^2\varrho K_0^*(0,\varrho,t_1)K_c(\varrho,0,t_2), \tag{2.10}$$

where $H_0 = \mathbf{p}^2$, $H = H_0 + V(\varrho)$, $H_c = H_0 + V_c(\varrho)$. The Green functions $K_c(\varrho_1,\varrho_2,t)$ and $K_0(\varrho_1,\varrho_2,t)$ are defined in [8] (see Eqs.(2.27), (2.24)). Carring out the calculations (some results obtained in Sections 4, 6 [8]) we find

for the spectral probability of radiation

$$\frac{dw}{d\omega} = \frac{\alpha}{\pi\omega}\sum_{k=1}^{4}\left[-r_1\text{Im } F_k^{(1)} + r_2\text{Re } F_k^{(2)}\right];$$
$$F_1^{(m)} = F_3^{(m)} = \int_0^\infty dt_1\int_0^T dt_2 e^{-it_1}\left[(t_1+t_2)^{-m}e^{-it_2} - N_1^m e^{-i\kappa t_2}\right],$$
$$F_2^{(m)} = \int_0^T dt(T-t)\left[(t-i0)^{-m}e^{-it} - N_2^m e^{-i\kappa t}\right],$$
$$F_4^{(m)} = \int_0^\infty dt_1\int_0^\infty dt_2 e^{-i(t_1+t_2)}\left[(t_1+t_2+T)^{-m}e^{-iT} - N_4^m e^{-i\kappa T}\right] \tag{2.11}$$

where

$$N_1 = \frac{\nu}{\sinh\nu t_2 + \nu t_1\cosh\nu t_2}, \quad N_2 = \frac{\nu}{\sinh\nu(t-i0)},$$
$$N_4 = \frac{\nu}{(1+\nu^2t_1t_2)\sinh\nu T + \nu(t_1+t_2)\cosh\nu T}, \quad \nu = \frac{1+i}{\sqrt{2}}\nu_0. \tag{2.12}$$

Note that in left-hand side of formula (2.11) m is an index, while in right-hand side m is a degree of a relevant functions. The functions $F_k^{(1,2)}$, $1 \le k \le 4$ are respectively the contributions of four domains of integration over t_1 and t_2 (see [8], section 4):

1. $t_1 \le 0,\ 0 \le t_2 \le T$;
2. $0 \le t_1 \le T,\ 0 \le t_2 \le T$;
3. $0 \le t_1 \le T,\ t_2 \ge T$;
4. $t_1 \le 0, t_2 \ge T$;

in two more domains $t_{1,2} \le 0$ and $t_{1,2} \ge T$ an electron is moving entirely free and there is no contribution from these domains.

Rearranging the subtraction terms in Eqs.(2.11), (2.12) we present the spectral probability of radiation as

$$\frac{dw}{d\omega} = \frac{\alpha}{\pi\omega}\sum_{k=1}^{5}\left[-r_1\text{Im } J_k^{(1)} + r_2\text{Re } J_k^{(2)}\right];$$
$$J_1^{(m)} = J_3^{(m)} = \int_0^\infty dt_1\int_0^T dt_2 e^{-i(t_1+\kappa t_2)}\left[(t_1+t_2)^{-m} - N_1^m\right],$$

$$J_2^{(m)} = \int_0^T dt(T-t)\mathrm{e}^{-i\kappa t}\left[t^{-m} - N_2^m\right],$$

$$J_4^{(m)} = \int_0^\infty dt_1 \int_0^\infty dt_2 \mathrm{e}^{-i(t_1+t_2+\kappa T)}\left[(t_1+t_2+T)^{-m} - N_4^m\right]$$

$$J_5^{(m)}(T) = 2\int_0^\infty dt_1 \int_0^T dt_2 \frac{\mathrm{e}^{-it_1}}{(t_1+t_2)^m}\left(\mathrm{e}^{-it_2} - \mathrm{e}^{-i\kappa t_2}\right)$$

$$+ \int_0^T dt \frac{(T-t)}{(t-i0)^m}\left(\mathrm{e}^{-it} - \mathrm{e}^{-i\kappa t}\right)$$

$$+ \int_0^\infty dt_1 \int_0^\infty dt_2 \frac{\mathrm{e}^{-i(t_1+t_2)}}{(t_1+t_2+T)^m}\left(\mathrm{e}^{-iT} - \mathrm{e}^{-i\kappa T}\right). \qquad (2.13)$$

The functions $J_5^{(m)}(T)$ we split into two parts:

$$J_5^{(m)}(T) = J_5^{(m)}(\infty) + j_5^{(m)}. \qquad (2.14)$$

Here $J_5^{(m)}(\infty)$ is the sum of the two first terms in the expression for $J_5^{(m)}(T)$ where the upper limit of integration T is substituted $T \to \infty$. So, the expression for $j_5^{(m)}$ is

$$j_5^{(m)}(T) = 2\int_0^\infty dt_1 \int_T^\infty dt_2 \frac{\mathrm{e}^{-it_1}}{(t_1+t_2)^m}\left(\mathrm{e}^{-it_2} - \mathrm{e}^{-i\kappa t_2}\right)$$

$$+ \int_T^\infty dt \frac{(T-t)}{(t-i0)^m}\left(\mathrm{e}^{-it} - \mathrm{e}^{-i\kappa t}\right)$$

$$+ \int_0^\infty dt_1 \int_0^\infty dt_2 \frac{\mathrm{e}^{-i(t_1+t_2)}}{(t_1+t_2+T)^m}\left(\mathrm{e}^{-iT} - \mathrm{e}^{-i\kappa T}\right) \qquad (2.15)$$

In the two-fold integrals in expressions for $J_5^{(m)}(\infty)$ and $j_5^{(m)}$ we replace the variables to $t = t_1 + t_2$ and t_2 take integrals over t_2. After this the expression for $J_5^{(1)}(\infty)$ contains the integral

$$\int_0^\infty \frac{dt}{t-i0}\left(\mathrm{e}^{-it} - \mathrm{e}^{-i\kappa t}\right)$$

$$= \int_0^\infty \frac{dt}{t}(\cos t - \cos \kappa t) + \frac{1}{2}\int_{-\infty}^\infty \frac{dt}{t-i0}\left(\mathrm{e}^{-it} - \mathrm{e}^{-i\kappa t}\right). \qquad (2.16)$$

The first term on the right-hand side is the Froullani integral equal to $\ln\kappa$. In the second term the integration contour can be closed in the lower half-plane,

where the integrand has no singularities, so that this integral vanishes. Using the above analysis we have

$$-\mathrm{Im}\, J_5^{(1)}(\infty) \equiv J_{tr}^{(1)} = 2\int_0^\infty \frac{dt}{t}\left[t\sin t - \frac{1}{\kappa-1}(\cos t - \cos\kappa t)\right] -$$

$$\int_0^\infty dt\,(\sin t - \sin\kappa t) = 1 + \frac{1}{\kappa} - \frac{2}{\kappa-1}\ln\kappa \qquad (2.17)$$

Similarly we have

$$\mathrm{Re}\, J_5^{(2)}(\infty) \equiv J_{tr}^{(2)} = 2\int_0^\infty \frac{dt}{t^2}\left[t\cos t - \frac{1}{\kappa-1}(\sin\kappa t - \sin t)\right] -$$

$$\int_0^\infty dt\,(\cos t - \cos\kappa t) = \left(1 + \frac{2}{\kappa-1}\right)\ln\kappa - 2, \qquad (2.18)$$

where integration by parts is fulfilled.

It is easy to check directly that the sum of the terms in $j_5^{(m)}$ which don't contain the parameter κ vanishes. Because of this we can write $j_5^{(m)}$ in the form

$$j_5^{(m)} = \int_T^\infty \frac{dt}{t^m}\left[\frac{2i}{\kappa-1}E_-(t) + (T-t)E_+(t)\right]$$

$$E_\pm(t) = \mathrm{e}^{-i\kappa t} \pm \mathrm{e}^{-i(t+(\kappa-1)T)} \qquad (2.19)$$

Integrals in (2.19) can be expressed in terms of integral sine $\mathrm{si}(x)$ and integral cosine $\mathrm{ci}(x)$ (see [9]):

$$-\mathrm{Im}\, j_5^{(1)} = \frac{2}{\kappa-1}A(\kappa,T) - TB_1(\kappa,T) - \left(1+\frac{1}{\kappa}\right)\cos\kappa T$$

$$\mathrm{Re}\, j_5^{(2)} = -\left(1+\frac{2}{\kappa-1}\right)A(\kappa,T) + TB_2(\kappa,T) + 2\cos\kappa T, \quad (2.20)$$

where

$$A(\kappa,T) = \mathrm{ci}(\kappa T) - \cos\varphi\,\mathrm{ci}(T) + \sin\varphi\,\mathrm{si}(T),$$

$$B_1(\kappa,T) = \mathrm{si}(\kappa T) + \cos\varphi\,\mathrm{si}(T) + \sin\varphi\,\mathrm{ci}(T),$$

$$B_2(\kappa,T) = \kappa\,\mathrm{si}(\kappa T) + \cos\varphi\,\mathrm{si}(T) + \sin\varphi\,\mathrm{ci}(T), \quad \varphi = (\kappa-1)T. \ (2.21)$$

In the expression for the spectral probability of radiation in form (2.13) in the limit $\nu_0 T \ll 1$ contribution of terms J_k^m (k=1,2,3) becomes small (it is proportional to powers of $\nu_0 T$). Moreover, for $\kappa T \geq 1$ the main contribution gives term J_5^m which is ν-independent. This term describes *transition radiation* (see (2.14), (2.18), (2.20), (2.21)). The term $J_{tr}^{(2)}$ is known expression for probability of transition radiation on one boundary, while $J_5^{(2)}$ as a whole describes the transition radiation on the plate with two boundaries (in the frame of the classical electrodynamics) and coincides with corresponding results in the transition radiation theory (see e.g.[10]). Our complete result in this case gives the probability of transition radiation in high-energy quantum electrodynamics.

For numerical calculations it is desirable to improve a convergence of the integrals in (2.13). For example, in the integral $J_1^{(2)}$ we rotate the integration contour over t_1 at an angle $-\pi/2$ and pass on to the variable $-it_1$. Then we have

$$J_1^{(2)} = i\int_0^{\infty} dt_1 \exp(-t_1) \int_0^T dt_2 \left[\frac{1}{(t_1+it_2)^2} - G_1^2\right] \exp(-i\kappa t_2),$$
$$G_1 = \frac{1}{t_1 \cosh \nu t_2 + (i/\nu)\sinh \nu t_2},\quad \nu = \frac{1+i}{\sqrt{2}}\nu_0,\quad \frac{i}{\nu} = \frac{1+i}{\sqrt{2}}\frac{1}{\nu_0}. \quad (2.22)$$

Taking integral over t_1 we obtain representation of $J_1^{(2)}$ as a single integral which is more convenient for numerical calculations

$$J_1^{(2)} = i\int_0^T dt \exp(-i\kappa t)\left\{\exp(it)\mathrm{Ei}(-it) - \frac{i}{t}\right.$$
$$\left. - \frac{1}{\cosh^2 \nu t}\left[\exp\left(\frac{i \tanh \nu t}{\nu}\right)\mathrm{Ei}\left(-\frac{i\tanh \nu t}{\nu}\right) - \frac{i\nu}{\tanh \nu t}\right]\right\}, \quad (2.23)$$

where $\mathrm{Ei}(z)$ is the exponential integral function defined as in [9]. In calculations one has to use appropriate branch of the function $\mathrm{Ei}(z)$ in the complex plane.

In the integral $J_4^{(2)}$ we substitute $t_1 \to -it_1$ and $t_2 \to -it_2$ and then replace the variables $t = t_1 + t_2, x = t_2$. The result is

$$J_4^{(2)} = \exp(-i\kappa T)\int_0^{\infty} dt \exp(-t)\int_0^t dx \left[\frac{1}{(t+iT)^2} - G_4^2\right],$$
$$G_4 = \frac{1}{(1-\nu^2 x(t-x))\,(i/\nu)\sinh \nu T + t\cosh \nu T}. \quad (2.24)$$

We consider first the case when LPM effect is weak ($\nu_1 \ll 1$). We assume here that condition $\nu_1(\omega_p) \geq 1$ (definition of ω_p see in (2.2)) is fulfilled, that

is in the region where $\nu_1 \ll 1$ one has $\omega \gg \omega_p$ and effects of the polarization of a medium are negligible. This is true for high energies ($\varepsilon \geq 10$ GeV). Then for thickness $T \ll 1/\nu_1$ the transverse shift of the projectile due to the multiple scattering in a target as a whole have no influence on coherent effects defined by the phase $\phi = \omega l(1-\mathbf{nv})$ in the factor $\exp(-i\phi)$. Indeed, for the projectile traversing a target in the case $\nu_1 T \ll 1$ an increment of the phase ϕ is small

$$\Delta\phi \sim \omega l \vartheta_s^2 \sim \omega l \frac{\nu_1^2 T}{\gamma^2} \sim \nu_1^2 T^2 \ll 1 \quad (2.25)$$

The angle of multiple scattering ϑ_s is small also comparing with an characteristic angle of radiation $1/\gamma$ ($\gamma^2\vartheta_s^2 = \nu_1^2 T \ll 1$). So, in the case $\nu_1 \ll 1$, $\nu_1 T \ll 1$ the radiation originates on separate atoms of a target and an interference on target boundaries is defined by the value $\omega l(1-v) = T$. At $T \ll 1$ this interference is weak, while at $T \gg 1$ there is a damping of the interference terms due to integration over photon emission angles. Expanding over ν_1 in (2.13) we obtain ($\kappa = 1$):

$$J^{(2)}(T) \equiv \mathrm{Re}\sum_{k=1}^{4} J_k^{(2)}(T) \simeq \frac{\nu_1^2 T}{3}\left[1 - 3T\int_1^{\infty}\frac{(x-1)^2}{x^3}\sin(xT)dx\right]$$
$$= \frac{\nu_1^2 T}{3}\left[1 + 3T\left(\left(1-\frac{T^2}{2}\right)\mathrm{si}(T) - 2T\mathrm{ci}(T) + \frac{3}{2}\sin T - \frac{T}{2}\cos T\right)\right] \quad (2.26)$$

For case $T \ll 1$

$$J^{(2)}(T) \simeq \frac{\nu_1^2 T}{3}\left[1 - \frac{3\pi}{2}T + 6T^2\left(\ln\frac{1}{T} + 1 - C\right)\right], \quad (2.27)$$

and for case $T \gg 1$

$$J^{(2)}(T) \simeq \frac{\nu_1^2 T}{3}\left(1 + 6\frac{\cos T}{T^2}\right). \quad (2.28)$$

Thus, in the case $\nu_1 \ll 1$, $\nu_1 T \ll 1$ the probability of radiation is defined by Bethe-Heitler formula both for $T \ll 1$ and for $T \gg 1$. However, for $T \sim 1$ the interference on the target boundaries is essential. If we present as above $J^{(2)}$ as product of $\nu_1^2 T/3$ (Bethe-Heitler formula) and some interference factor, then this factor attains 0.53 at $T = 0.32$ (minimum of the interference factor) and 1.33 at $T = 1.84$ (maximum of the interference factor).

When the parameter $\nu_1 T$ is large ($\nu_1 \ll 1$, $\nu_1 T \gg 1$) the radiation is formed inside a target and the interference terms are damping exponentially.

In this case formulae derived in [8] for thick target are applicable. In this case value of separate terms in the sum for Re $J^{(2)}(T)$ could strongly oscillate:

$$
\begin{aligned}
&\mathrm{Re}\, J_1^{(2)}(T) = \mathrm{Re}\, J_3^{(2)}(T)) \simeq \mathrm{Re}\, J_1^{(2)}(\infty) - \int_0^{\infty} dt_1 \int_T^{\infty} dt_2 \frac{\cos(t_1+t_2)}{(t_1+t_2)^2} \\
&= \mathrm{Re}\, J_1^{(2)}(\infty) - \Delta(T), \quad \mathrm{Re}\, J_2^{(2)}(T) \simeq \mathrm{Re}\, J_2^{(2)}(\infty) + \Delta(T), \\
&\mathrm{Re}\, J_4^{(2)}(T) \simeq \int_0^{\infty} dt_1 \int_0^{\infty} dt_2 \frac{\cos(t_1+t_2+T)}{(t_1+t_2+T)^2} = \Delta(T), \\
&\Delta(T) = \int_T^{\infty} \frac{(t-T)}{t^2} \cos T dt \simeq -\int_T^{\infty} \frac{\sin t}{t^2} dt \simeq -\frac{\cos T}{T^2}. \qquad (2.29)
\end{aligned}
$$

It is seen from (2.29) that in the sum for $J^{(2)}(T)$ the contribution of terms $\Delta(T)$ is canceled exactly. In the considered case ($\nu_1 \ll 1$) the value $J^{(2)}(\infty)$ gives the formula Bethe-Heitler with corresponding corrections. Remind that in the limit $\nu_1 \to 0$ the exact Bethe-Heitler formula can be obtained only if the terms $\propto 1/L$ are taken into account [8]. The expression for $J^{(2)}$ found in Sect.4 of [8] is

$$
J^{(2)}(\infty) = \frac{\nu_1^2 T}{3}\left(1 + \frac{1}{6L_1} - \frac{16\nu_1^4}{21}\right) - \frac{2\nu_1^4}{21}, \qquad (2.30)
$$

where L_1 and ν_1 are defined in (2.4) and (2.7).

We consider now the case when the LPM effect is strong ($\nu_0 \gg 1$) and the parameter $T \ll 1$ while the value which characterize the thickness of a target $\nu_0 T \sim 1$. Such situation is possible at $\omega \ll \varepsilon$. So, we can omit terms with $r_1 = \omega^2/\varepsilon^2$ and put $r_2 \simeq 2$. We expand the expressions for $J_k^{(2)}$ in a power series in $\dfrac{1}{\nu_0}$ and T including linear terms in $\dfrac{1}{\nu_0}$ and T. The resulting decompositions are

$$
\begin{aligned}
J_1^{(2)} = J_3^{(2)} &\simeq \ln\frac{\nu T}{\tanh \nu T} + i\left[\left(\frac{\tanh \nu T}{\nu} - T\right)\left(1 - C - i\frac{\pi}{2}\right)\right. \\
&\left. + \frac{\tanh \nu T}{\nu} \ln\frac{\nu}{\tanh \nu T} - T\ln\frac{1}{T} + \kappa\left(\frac{2}{\nu}\int_0^{\nu T} \frac{t dt}{\sinh 2t} - T\right)\right], \\
J_2^{(2)} &\simeq (1 + i\kappa T)\ln\frac{\sinh \nu T}{\nu T} + 2i\kappa\left(T - \frac{1}{\nu}\int_0^{\nu T} dt \coth t\right), \\
J_4^{(2)} &\simeq \exp(-i\kappa T)\left\{2\ln\tanh \nu T + \frac{2i}{\nu}\left[\coth \nu T \ln\frac{\nu}{\coth \nu T}\right.\right.
\end{aligned}
$$

$$
\begin{aligned}
&\left.- \tanh \nu T \ln\frac{\nu}{\tanh \nu T} + \frac{2}{\sinh 2\nu T}\left(1 - C - i\frac{\pi}{2}\right)\right] \\
&\left.- (1 + 2iT)\left(\ln T + C + i\frac{\pi}{2}\right) - 1 + iT\right\}. \qquad (2.31)
\end{aligned}
$$

The presented here expressions for functions $J_1^{(2)}$, $J_2^{(2)}$, $J_3^{(2)}$ are not valid in the case $\kappa T \geq 1$. However, in this case (under condition that $\nu_0 T \ll 1$) the contribution of terms $J_1^{(2)}$, $J_2^{(2)}$, $J_3^{(2)}$ is negligible both in the asymptotic expressions (2.31) and exact formulae (2.13). This means that expressions (2.31) may be used at any value κT when $\nu_0 \gg 1, T \ll 1$. Substituting obtained asymptotic decompositions into Eq.(2.13) we find for $\kappa T \ll 1$

$$
\begin{aligned}
&\frac{dw}{d\omega} = \frac{2\alpha}{\pi\omega}\left(J^{(2)} + J_5^{(2)}\right), \; J_5^{(2)} \simeq \frac{(\kappa-1)^2}{2}\left(\ln\frac{1}{T} + \frac{1}{2} - C\right), \\
&J^{(2)} \equiv \mathrm{Re}\sum_{k=1}^{4} J_k^{(2)} \simeq \mathrm{Re}\left\{\ln(\nu\sinh \nu T) - 1 - C - \kappa\frac{\pi T}{4}\right. \\
&+ \frac{2i}{\nu\tanh \nu T}\left[\ln(\nu\tanh \nu T) + 1 - C - \frac{i\pi}{2}\right] \\
&+ i\kappa T\left(\ln\frac{\cosh \nu T}{\tanh \nu T} - \frac{\nu T}{\tanh \nu T}\right) \\
&\left.+ \frac{i\kappa}{\nu}\int_0^{\nu T} dt\left(\frac{4t}{\sinh 2t} - \frac{t^2}{\sinh^2 t}\right)\right\}, \quad \nu = \exp\left(i\frac{\pi}{4}\right)\nu_0. \qquad (2.32)
\end{aligned}
$$

For a relatively thick target ($\nu_0 T \gg 1$) we have from (2.32)

$$
\begin{aligned}
J^{(2)} &\simeq \ln\nu_0 - 1 - C - \ln 2 + \frac{\sqrt{2}}{\nu_0}\left(\kappa\frac{\pi^2}{24} + \ln\nu_0 + 1 - C + \frac{\pi}{4}\right) \\
&+ \frac{\nu_0 T}{\sqrt{2}}\left(1 - \frac{\pi\kappa}{2\sqrt{2}\nu_0}\right) \qquad (2.33)
\end{aligned}
$$

Here the terms without T are the contribution of boundary photons (formula (4.14) of [8]) while the term $\propto T$ gives in (2.33) the probability of radiation inside target (with correction $\sim \kappa/\nu_0$ but without corrections $\sim 1/L$). The relative value of the last corrections at $\nu_0 \gg 1$ is (Eq.(2.45) of [8])

$$
r = \frac{1}{2L(\varrho_c)}\left(\ln 2 - C + \frac{\pi}{4}\right) \simeq \frac{0.451}{L(\varrho_c)}. \qquad (2.34)
$$

In the limiting case when a target is very thin and $\nu_0 T \ll 1$ but when $\nu_0^2 T \gg 1$ we have from (2.32)

$$J^{(2)} \simeq \left(1+\frac{2}{\nu_0^2 T}\right)\left[\ln(\nu_0^2 T)+1-C\right]-2+\delta,$$

$$\delta = \frac{(\nu_0 T)^4}{180}+\frac{2(\nu_0 T)^2}{45}T\left(\ln \nu_0^2 T - C\right)-\frac{\kappa T}{6}(\nu_0 T)^2. \qquad (2.35)$$

The terms without δ in this expression coincide with formula (5.15) of [8] (up to terms $\propto C/L_t$).

In the photon energy region where $\nu_0 T \ll 1$ the contribution of the terms $J_k^{(m)}$ (k=1,2,3) is very small ($\sim \delta$) and decreases with photon energy reduction ($\propto \omega$), so that in the spectral distribution of radiation only the terms $J_4^{(m)}, J_5^{(m)}$ contribute. We consider now the function $J_4^{(2)}$ in the case when $(1+\nu_0)T \ll 1$ and the parameter $\nu_0^2 T$, which characterizes the mean square angle of the multiple scattering in a target as a whole, has an arbitrary value. Under the mentioned conditions the function N_4 in (2.12) may be written as

$$N_4^2 \simeq \left(\nu^2 T t_1 t_2 + t_1 + t_2\right)^{-2} = -\int_0^\infty dx x \exp\left[-ix\left(\nu^2 T t_1 t_2 + t_1 + t_2\right)\right]. \qquad (2.36)$$

Substituting this expression in (2.13) we find

$$J_4^{(2)} e^{i\kappa T} \simeq \int_0^\infty dx x \int_0^\infty dt_1 \int_0^\infty dt_2 \exp\left(-i(1+x)(t_1+t_2)\right) \times \left[\exp\left(-ix\nu^2 T t_1 t_2\right)-1\right]. \qquad (2.37)$$

Making the substitution of variables

$$t_{1,2} \to -it_{1,2}, \quad x \to \frac{x}{t_1 t_2}$$

we obtain

$$J_4^{(2)} e^{i\kappa T} = \int_0^\infty dx x \int_0^\infty \frac{dt_1}{t_1^2}\int_0^\infty \frac{dt_2}{t_2^2}\exp\left(-\left(1+\frac{x}{t_1 t_2}\right)(t_1+t_2)\right) \times \left[1-\exp\left(-x\nu_0^2 T\right)\right] = 4\int_0^\infty dx K_1^2\left(2\sqrt{x}\right)\left[1-\exp\left(-x\nu_0^2 T\right)\right]$$
$$= 2\int_0^\infty d\varrho\varrho K_1^2(\varrho)\left[1-\exp\left(-k\varrho^2\right)\right], \quad 4k=\nu_0^2 T, \qquad (2.38)$$

where $K_1(\varrho)$ is the modified Bessel function. Formula (2.38) corresponds at $\kappa = 1$ to result for a thin target obtained in [8] (see Eq.(5.7)) without terms $\propto 1/L$. Since the dependence on the parameter κ is contained in (2.38) as a common phase multiplier $\exp(-i\kappa T)$, one can write more accurate expression for $J_4^{(2)}$ (with terms $\propto 1/L$) using the results of [8](see Eq.(5.9)):

$$J_4^{(2)} = 2e^{-i\kappa T}\int_0^\infty d\varrho\varrho K_1^2(\varrho)\left[1-\exp\left(-V(\varrho)T\right)\right],$$

$$V(\varrho)T = \frac{\pi Z^2\alpha^2 n l \varrho^2}{m^2}\left(\ln\frac{4a_{s2}^2}{\lambda^2\varrho^2}-2C\right). \qquad (2.39)$$

For the case $\nu_0^2 T \gg 1$ it has the form

$$e^{i\kappa T} J_4^{(2)} = \left(1+\frac{1}{2k}\right)\left[\ln 4k + 1 - C\right] - 2 + \frac{C}{L_t},$$

$$k = \frac{\pi Z^2\alpha^2 nl}{m^2}L_t, \quad L_t = \ln\frac{4a_{s2}^2}{\lambda_c^2\varrho_t^2}-2C, \quad k(\varrho_t)\varrho_t^2 = 1 \qquad (2.40)$$

In the case when parameter k is not very high one has to use an exact expression found in [8] (formula (5.7)). For $k \ll 1$ one can expand the exponent in the integrand of (2.39). Then we find

$$e^{i\kappa T} J_4^{(2)} = \frac{\nu_1^2 T}{3}\left(1+\frac{1}{6L_1}\right), \quad \nu_1^2 T = \frac{4\pi Z^2\alpha^2 nl}{m^2}L_1. \qquad (2.41)$$

At $\kappa T \ll 1$ the spectral distribution of probability is

$$\frac{dw}{d\omega} = \frac{2\alpha}{3\pi\omega}\nu_1^2 T\left(1+\frac{1}{6L_1}\right)\left(1-\frac{\omega}{\varepsilon}\right). \qquad (2.42)$$

This is the Bethe-Heitler formula for not very hard photons (terms $\propto \left(\frac{\omega}{\varepsilon}\right)^2$ are omitted).

When a photon energy decreases, the parameter κ increases as well as the combination $\kappa T \propto 1/\omega$, while the value $(\nu_0 T)^2$ decreases $\propto \omega$. Just this value defines an accuracy of Eq.(2.39). Using Eqs.(2.14)-(2.21) at $T \ll 1, \kappa T \geq 1$ we find for the probability of the transition radiation following expression

$$\frac{dw_{tr}}{d\omega} \simeq \frac{2\alpha}{\pi}\left\{\left(1+\frac{2}{\kappa-1}\right)\left[\ln\kappa - \mathrm{ci}(\kappa T) + \cos(\kappa T)(\ln T + C) + \frac{\pi}{2}\sin(\kappa T)\right] + \kappa T\,\mathrm{si}(\kappa T) - 4\sin^2\frac{\kappa T}{2}\right\}. \qquad (2.43)$$

In the limiting case $\kappa T \gg 1$ the probability (2.43) turns into standard probability of the transition radiation with oscillating additions

$$\frac{dw_{tr}}{d\omega} = \frac{2\alpha}{\pi}\left[J_{tr} + \cos(\kappa T)\left(\ln T + C + 1\right) + \frac{\pi}{2}\sin(\kappa T)\right] \quad (2.44)$$

Note, that there is a qualitative difference in a behaviors of interference terms in Eqs.(2.28) and (2.44). In the former an amplitude of oscillation with ω increase decreases as $1/\omega^2$ whilst in the latter the corresponding amplitude weakly (logarithmically) increases with ω decrease.

From the above analysis follows that in the case when $\nu_0 T \ll 1$ ($\nu_0 \gg 1$) the spectral distribution of probability of radiation with the polarization of a medium taken into account has the form

$$\frac{dw}{d\omega} = \frac{dw_{tr}}{d\omega} + \cos(\kappa T)\frac{dw_{th}}{d\omega}, \quad (2.45)$$

where $dw_{th}/d\omega$ is the spectral distribution of probability of radiation in a thin target without regard for the polarization of a medium. In the case $4k = \nu_0^2 T \gg 1$ the probability $dw_{th}/d\omega$ is defined by Eq.(2.40) and for the case $k \ll 1$ it is defined by Eq.(2.41). More accurate representation of the probability of radiation $dw_{th}/d\omega$ may be obtained using Eq.(2.39). It follows from Eqs.(2.44) and (2.45) that if we make allowance for multiple scattering at $\kappa T \gg 1$ this results in decreasing of oscillations of the transition radiation probability by magnitude of the bremsstrahlung probability in a thin target.

3 A qualitative behavior of the spectral intensity of radiation

We consider the spectral intensity of radiation for the energy of the initial electrons when the LPM suppression of the intensity of radiation takes place for relatively soft energies of photons: $\omega \leq \omega_c \ll \varepsilon$:

$$\nu_1(\omega_c) = 1, \quad \omega_c = \frac{16\pi Z^2\alpha^2}{m^2}\gamma^2 n \ln\frac{a_{s2}}{\lambda_c}, \quad (3.1)$$

see Eqs.(2.4), (2.6), (2.7) and (2.9). This situation corresponds to the experimental conditions.

A ratio of a thickness of a target and the formation length of radiation (2.8) is an important characteristics of the process. This ratio may be written

as

$$\beta(\omega) = T(\nu_0 + \kappa) \simeq T_c\left[\frac{\omega}{\omega_c} + \sqrt{\frac{\omega}{\omega_c}} + \frac{\omega_p^2}{\omega\omega_c}\right],$$

$$T = \frac{l\omega}{2\gamma^2}, \quad \omega_p = \omega_0\gamma, \quad T_c \equiv T(\omega_c) \simeq \frac{2\pi}{\alpha}\frac{l}{L_{rad}}, \quad (3.2)$$

where we put that $\nu_0 \simeq \sqrt{\frac{\omega_c}{\omega}}$. Below we assume that $\omega_c \gg \omega_p$ which is true under the experimental conditions.

If $\beta(\omega_c) = 2T_c \ll 1$ then at $\omega = \omega_c$ a target is thin and the Bethe-Heitler spectrum of radiation is valid at $\omega \leq \omega_c$ in accordance with Eq.(2.39) since $4k = \nu_0^2 T = T_c \ll 1$. This behavior of the spectral curve will continue with ω decrease until photon energies where a contribution of the transition radiation become essential. In this case the spectral distribution of radiation has the form (2.45) for all ω

$$\frac{dw}{d\omega} = \frac{dw_{tr}}{d\omega} + \cos(\kappa T)\frac{dw_{BH}}{d\omega}, \quad (3.3)$$

Since for soft photons ($\omega \ll \varepsilon$)

$$\frac{dI}{d\omega} = \frac{2\alpha}{\pi}\left[J_5^{(2)} + \frac{T_c}{3}\left(1 + \frac{1}{6L_1}\right)\cos(\kappa T)\right] \quad (3.4)$$

and $T_c/3 \ll 1$ a contribution of the transition radiation become visible already at $\kappa T \ll 1$. For $\omega > \omega_c$ ($T_c \ll 1$) the probability of radiation is defined by (2.26)-(2.28). In this case a considerable distinction from Bethe-Heitler formula will be in the region $\omega \sim \omega_c/T_c$.

If $\beta(\omega_c) \gg 1$ ($T_c \gg 1$) then at $\omega \geq \omega_c$ a target is thick and one has the LPM suppression for $\omega \leq \omega_c$. There are two opportunities depending on the minimal value of the parameter β.

$$\beta_m \simeq \frac{3}{2}T_c\sqrt{\frac{\omega_1}{\omega_c}}, \quad \omega_1 = \omega_p\left(\frac{4\omega_p}{\omega_c}\right)^{1/3}, \quad \beta_m \simeq 2T_c\left(\frac{\omega_p}{\omega_c}\right)^{2/3}. \quad (3.5)$$

If $\beta_m \ll 1$ then for photon energies $\omega > \omega_1$ it will be ω_2 such that

$$\beta(\omega_2) = 1, \quad \omega_2 \simeq \frac{\omega_c}{T_c^2} \quad (3.6)$$

and for $\omega < \omega_2$ the thickness of a target becomes smaller than the formation length of radiation so that for $\omega \ll \omega_2$ the spectral distribution of the radiation intensity is described by formula (2.39). In this case for $4k = \nu_0^2 T \simeq T_c \gg 1$ one has (2.31). Under conditions $\kappa T \ll 1, \omega < \omega_2$ the spectral intensity of radiation is independent of photon energy ω. It should be noted that due to smallness of the coefficients in expression for δ (2.35), such behaviors of the spectral curve begins at $\omega < \omega_{th} = 4\omega_2 \simeq 4\omega_c/T_c^2$. Such behavior of the spectral curve will continue until photon energies where one has to take into account the polarization of a medium and connected with it a contribution of the transition radiation.

As well known, for soft photons ($\omega \ll \varepsilon$) the Bethe-Heitler formula for the spectral intensity of radiation doesn't depend on a photon energy as well. So, the ratio of these spectral intensities (see Eqs. (2.39) and (2.42)) is an important characteristics of the phenomenon under consideration:

$$R = \frac{dI_{th}}{d\omega}/\frac{dI_{BH}}{d\omega} = \frac{6}{T_c}\left(1 + \frac{1}{6L_1}\right)^{-1} \int_0^\infty d\varrho \varrho K_1^2(\varrho)\left[1 - \exp\left(-V(\varrho)T\right)\right] \tag{3.7}$$

For $T_c \ll 1$ one has $R = 1$ and for $T_c \gg 1$ one has using expression (2.40)

$$R \simeq \frac{3}{T_c}\left(1 + \frac{1}{2k}\right)\left[\ln 4k + 1 - C\right] - 2 + \frac{C}{L_t} \tag{3.8}$$

For estimates one can put with a good accuracy $4k \simeq T_c$ and $L_t \simeq L_1$.

At $\beta_m \gg 1$ a target remains thick for all photon energies and radiation is described in details by formulae of Sections 2 and 3 of [8] where comparison with experimental data was carried out as well.

For very high energies when the LPM effect becomes significant at $\omega \sim \varepsilon$ Eq.(3.1) should be substituted by

$$\omega_c = \frac{16\pi Z^2\alpha^2}{m^2}\gamma^2\left(1 - \frac{\omega_c}{\varepsilon}\right) n \ln\frac{a_{s2}}{\lambda_c}, \tag{3.9}$$

so that

$$\frac{\omega_c/\varepsilon}{(1 - \omega_c/\varepsilon)} = \frac{4\pi}{\alpha}\gamma\frac{\lambda_c}{L_{rad}} = r \tag{3.10}$$

It is evident that for $\omega < \omega_c$ the radiation losses diminish (for very rough estimation one can use as reduction factor $r/(1+r)$) and due to this the radiation length enlarges. Of course, for the electron energy $\varepsilon = 25$ GeV this effect is very weak (order of 1%). However, for very high energy it becomes quite sizable. For example, for the electron energy $\varepsilon = 500$ GeV this effect is of the order of 16%.

There is, in principle, an opportunity to measure the electron energy (in region of high energies) using the LPM effect. For this one can measure the spectral curve on a target with thickness a few percent of L_{rad} and compare the result with the theory prediction [8].

Existence of the plateau of the spectral curve in a region of photon energies where a target is thin was found in [11] within Migdal approach (quantum theory). Recently this item was discussed in [12] (in classical theory), [13] and [14].

4 Discussion and conclusions

In [8] the qualitative analysis of the data [5]-[7] was performed. It was noted that for targets with thickness $l \geq 2\%L_{rad}$ the formation length of radiation $l_f \ll l$ for any photon energy ω. So, these targets can be considered as thick targets. The gold targets with thickness $l = 0.7\%L_{rad}$ and $l = 0.1\%L_{rad}$ are an exception. We calculated energy losses spectra in these targets for the initial electron energy $\varepsilon = 25$ GeV and $\varepsilon = 8$ GeV. The characteristic parameters of radiation for these cases are given in Table.

Table: Characteristic parameters of the radiation process in gold with the thickness $l = 0.7\%L_{rad}$ and $l = 0.1\%L_{rad}$ all photon energies ω are in MeV

ε (GeV)	ω_c	ω_p	$T_c(0.7)$	$T_c(0.1)$	$\omega_1(0.7)$	$\beta_m(0.7)$	$\beta_m(0.1)$	ω_{th}
25	239	3.92	5.82	0.96	1.6	0.75	0.12	28
8	24.5	1.25	5.82	0.96	0.76	1.6	0.25	3.0

In Fig.1(a) results of calculations are given for target with a thickness $l = 0.7\%L_{rad}$ at $\varepsilon = 25$ GeV. The curves 1,2,3,4 present correspondingly the functions $J_1^{(2)}, J_2^{(2)}, J_3^{(2)}, J_4^{(2)}$ (2.13). At $\omega = 500$ MeV the value $\nu_1 T = \sqrt{\frac{\omega}{\omega_c}}T_c = 8.4 \gg 1$. The interference terms are exponentially small and one can use formulae for a thick target. In this case the parameter $\nu_1 = 0.69$ and contribution of boundary photons ($J_b = J_1^{(2)} + J_3^{(2)} + J_4^{(2)}$) is small ($J_b \simeq -\frac{2\nu_1^4}{21}$, see [8], Eq.(4.16)) and distinction Bethe-Heitler formula ($J_{BH} = T_c/3 = 1.94$) from Re $J^{(2)} =$ Re $(J_1^{(2)} + J_2^{(2)} + J_3^{(2)} + J_4^{(2)})$ is of the order 10% according with asymptotic damping factor $\left(1 - \frac{16\nu_1^4}{21}\right)$.

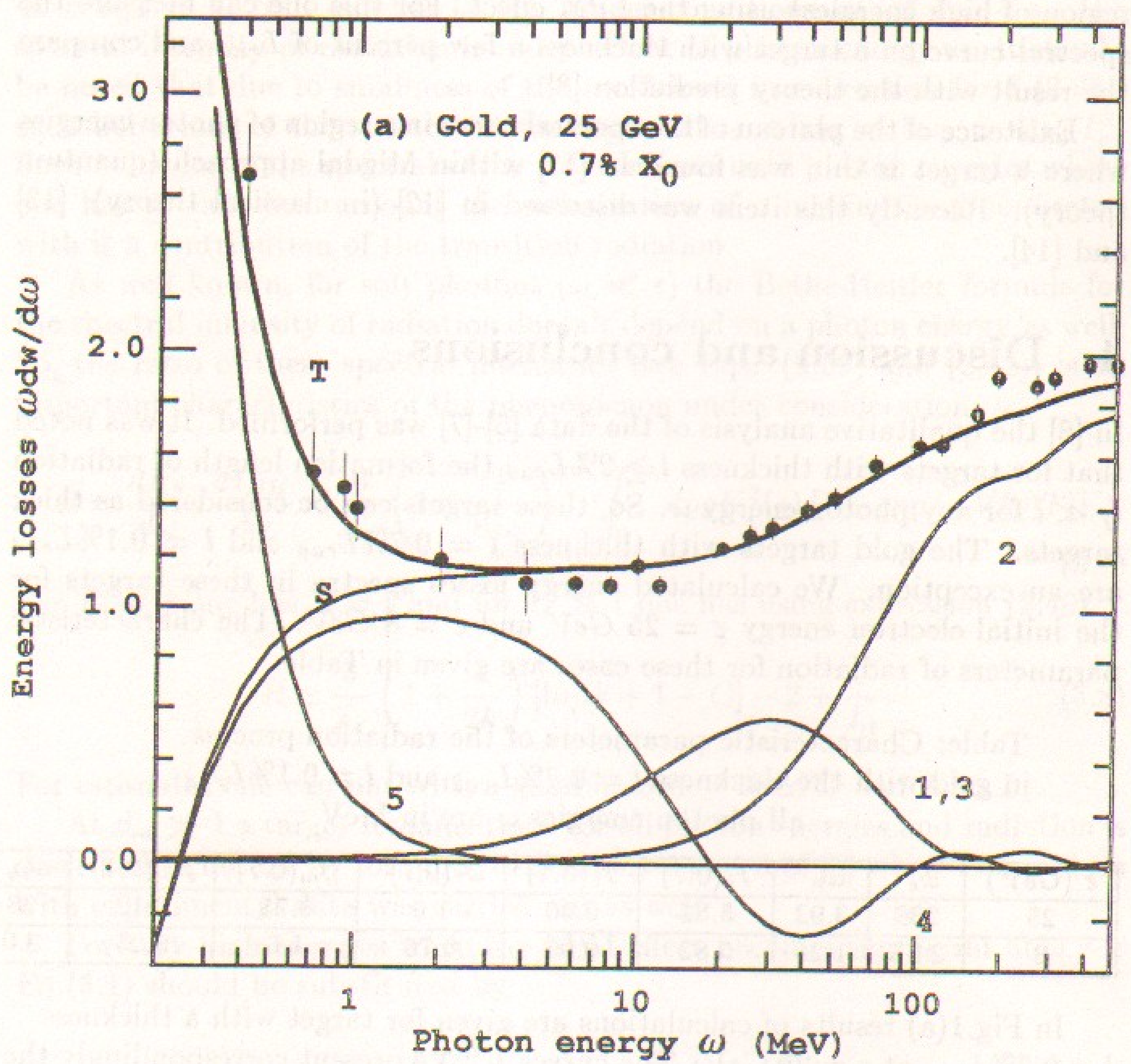


Fig.1(a)

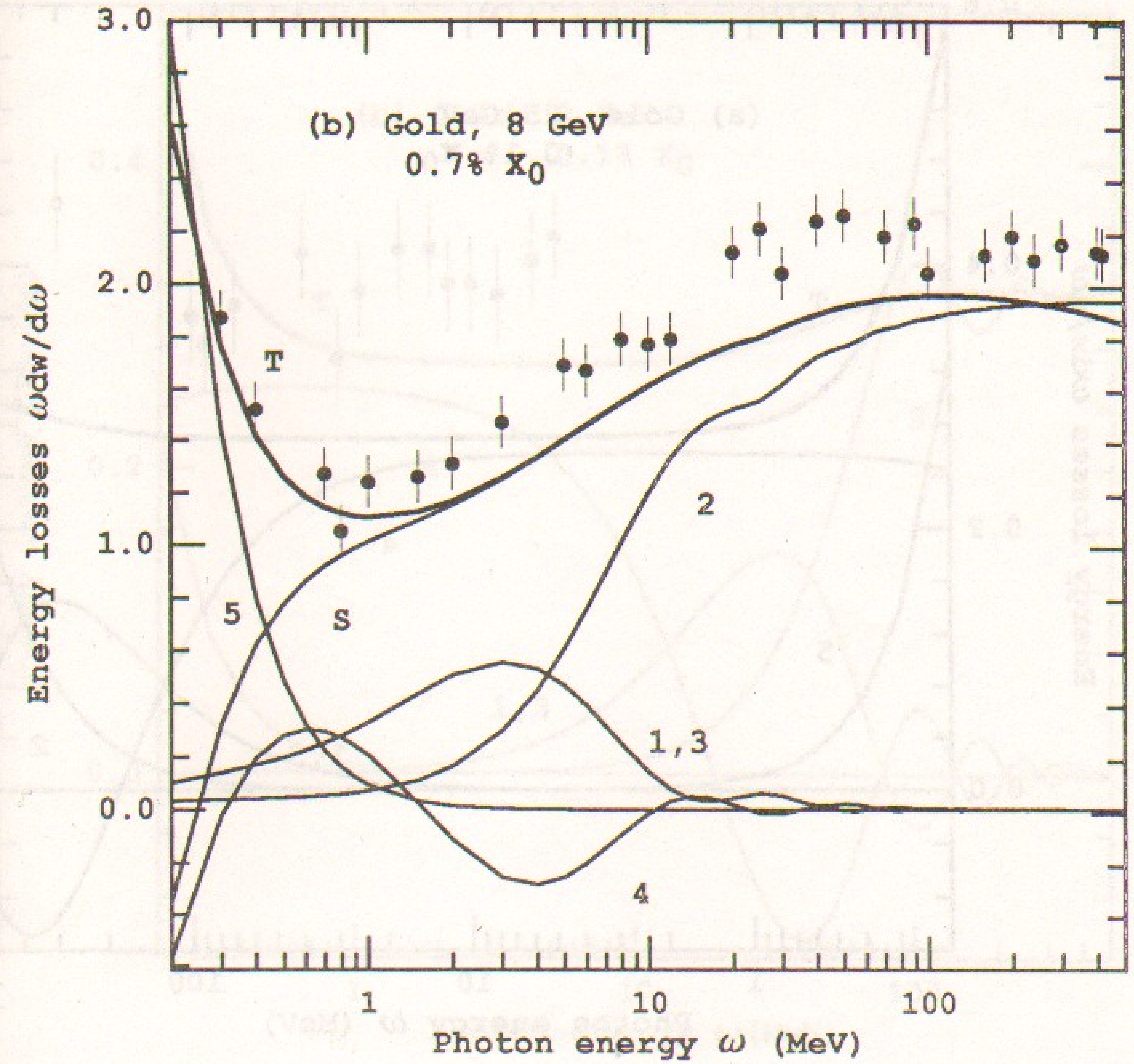


Fig.1(b)

Fig.1. The energy losses $\omega\frac{dW}{d\omega}$ in gold with thickness $l = 0.023\ mm$ in units $\frac{2\alpha}{\pi}$; ((a) is for the initial electrons energy $\varepsilon = 25\ GeV$ and (b) is for $\varepsilon = 8\ GeV$). The Coulomb corrections and the polarization of a medium are included. Curve 1 is the contribution of the term Re $J_1^{(2)}$ = Re $J_3^{(2)}$; curve 2 is the contribution of the term Re $J_2^{(2)}$; curve 4 is the contribution of the term Re $J_4^{(2)}$, all (2.13); curve S is the sum of the previous contributions Re $J^{(2)}$; curve 5 is the contribution of the boundary photons (2.20); curve T is the total prediction for the radiation energy losses. Experimental data from Fig.12 of [7].

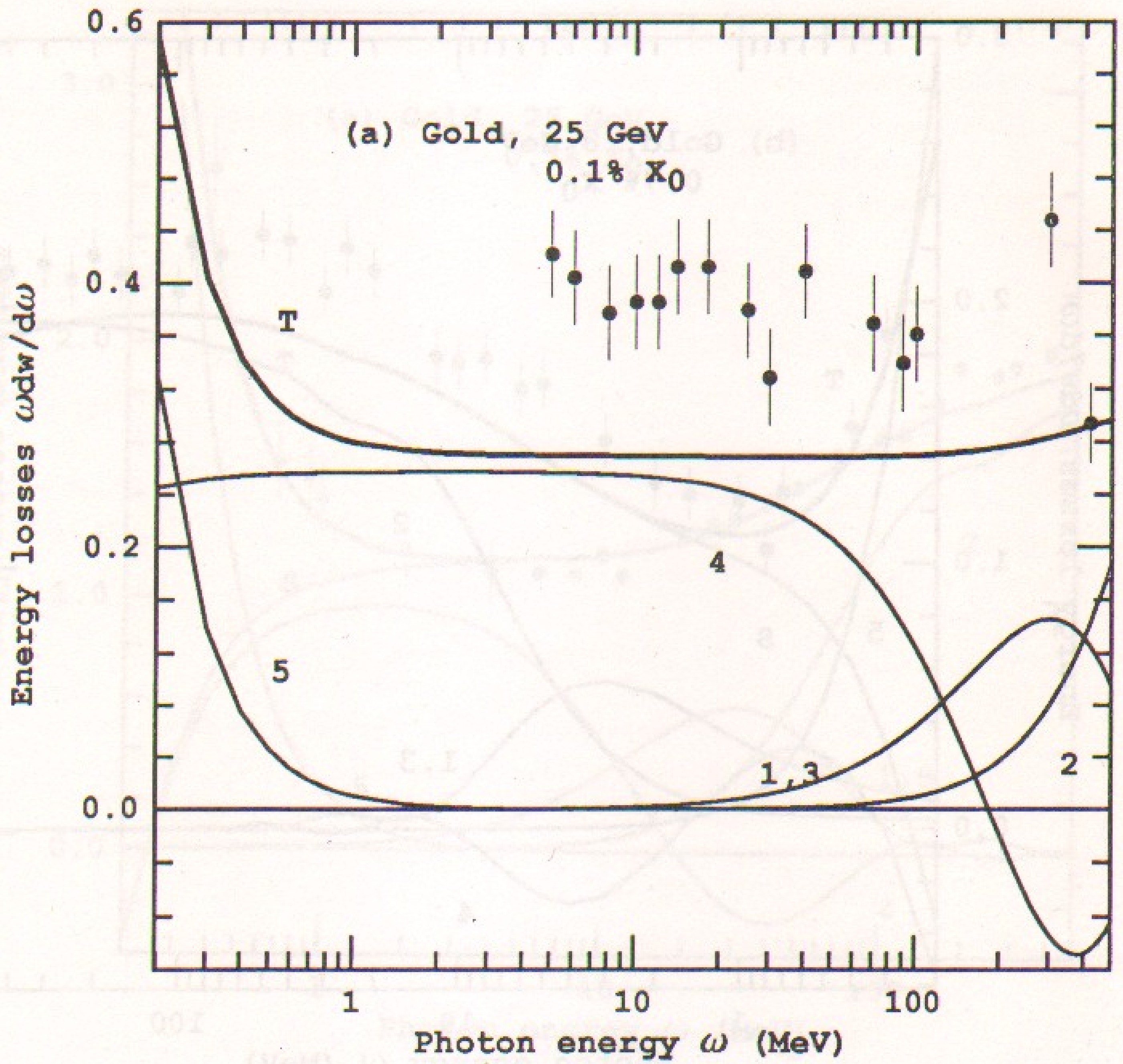


Fig.2(a)

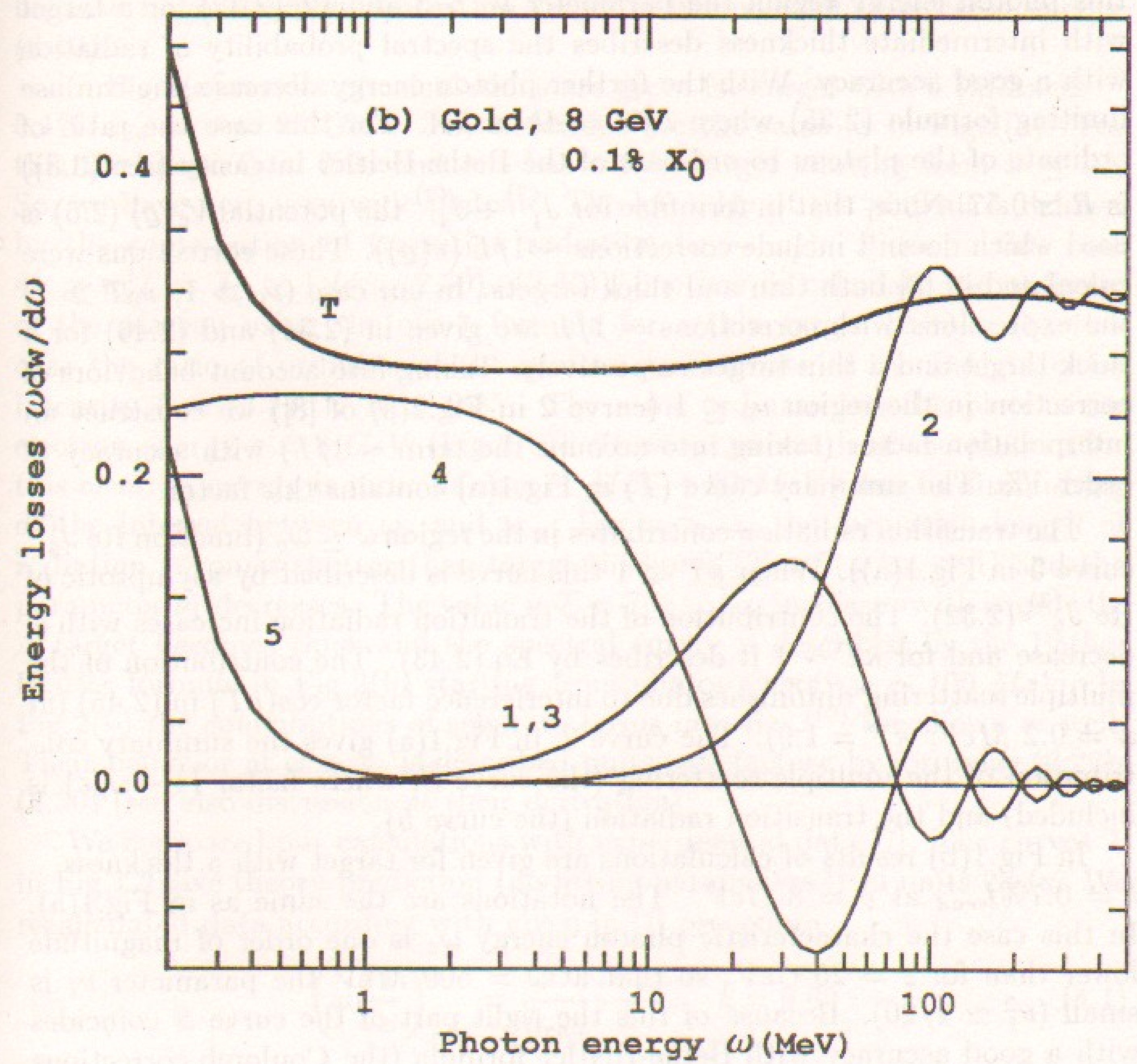


Fig.2(b)

Fig.2. The energy losses $\omega\frac{dW}{d\omega}$ in gold with thickness $l = 0.0038\ mm$ in units $\frac{2\alpha}{\pi}$. ((a) is for the initial electrons energy $\varepsilon = 25\ GeV$ and (b) is for $\varepsilon = 8\ GeV$). The Coulomb corrections and the polarization of a medium are included. Curve 1 is the contribution of the term Re $J_1^{(2)}$ = Re $J_3^{(2)}$; curve 2 is the contribution of the term Re $J_2^{(2)}$; curve 4 is the contribution of the term Re $J_4^{(2)}$, all (2.13); curve S is the sum of the previous contributions Re $J^{(2)}$; curve 5 is the contribution of the boundary photons (2.20); curve T is the total prediction for the radiation energy losses . Experimental data from Fig.13 of [7].

At $\omega < \omega_{th} \simeq 30\ MeV$ for the case $T_c \gg 1$ and $\beta_m < 1$ the spectral curve turns into plateau according with discussion in previous Section. In this photon energy region the parameter $\nu_0 > 3$ and Eq.(2.32) for a target with intermediate thickness describes the spectral probability of radiation with a good accuracy. With the further photon energy decrease one can use limiting formula (2.35) where $\nu_0^2 T = 4k \simeq 7.4$. For this case the ratio of ordinate of the plateau to ordinate of the Bethe-Heitler intensity (see (3.8)) is $R \simeq 0.57$. Note, that in formulae for $J_1^{(2)} \div J_4^{(2)}$ the potential $V_c(\varrho)$ (2.6) is used which doesn't include corrections $\sim 1/L$ ($v(\varrho)$). These corrections were calculated in [8] both thin and thick targets. In our case ($\nu_0 \gg 1$, $\nu_0 T \gg 1$) the expressions with corrections $\sim 1/L$ are given in (2.34) and (2.40) for a thick target and a thin target respectively. Taking into account behaviors of correction in the region $\nu_0 \leq 1$ (curve 2 in Fig.2(a) of [8]) we construct an interpolation factor (taking into account the term $\sim 1/L$) with accuracy of order 1%. The summary curve (T) in Fig.1(a) contains this factor.

The transition radiation contributes in the region $\omega \leq \omega_p$ (function Re $J_5^{(2)}$, curve 5 in Fig.1(a)). When $\kappa T \ll 1$ this curve is described by asymptotic of Re $J_5^{(2)}$ (2.32). The contribution of the transition radiation increases with ω decrease and for $\kappa T \sim 1$ it describes by Eq.(2.43). The contribution of the multiple scattering diminishes due to interference factor $\cos(\kappa T)$ in (2.45) (at $\omega = 0.2\ MeV$, $\kappa T = 1.9$). The curve T in Fig.1(a) gives the summary contribution of the multiple scattering (the curve S, where factor $(1 - \omega/\varepsilon)$ is included) and the transition radiation (the curve 5).

In Fig.1(b) results of calculations are given for target with a thickness $l = 0.7\% L_{rad}$ at $\varepsilon = 8\ GeV$. The notations are the same as in Fig.1(a). In this case the characteristic photon energy ω_c is one order of magnitude lower than for $\varepsilon = 25\ GeV$, so that at $\omega = 500\ MeV$ the parameter ν_1 is small ($\nu_1^2 \simeq 1/20$). Because of this the right part of the curve S coincides with a good accuracy with Bethe-Heitler formula (the Coulomb corrections are included). Note, that for this electron energy the effect of recoil (factor $(1 - \omega/\varepsilon)$) is more essential. Strictly speaking, a target with a thickness $0.7\% L_{rad}$ at $\varepsilon = 8\ GeV$ is not thin target for any photon energy ($\beta_m = 1.6$). However, for bremsstrahlung this target can be considered as a thin one for $\omega < \omega_{th} = 3\ MeV$. Since the polarization of a medium becomes essential in the same region ($\omega_p = 1.25\ MeV$), the interference factor $\cos(\kappa T)$ in (2.45) causes an inflection of the spectral curve S at $\omega \sim 1\ MeV$. The transition radiation grows from the same photon energy ω and because of this the total spectral curve T has a minimum at $\omega \simeq 1\ MeV$. As far as there is some interval of energies between ω_p and ω_{th} ($\omega_{th} - \omega_p \sim 3\ MeV$), this minimum

is enough wide. Moreover, the value of its ordinate coincide with a good accuracy with ordinate of the plateau of the spectral curve S in Fig.1(a) because bremsstrahlung on a thin target is independent of electron energy (2.40).

In Fig.2(a) results of calculations are given for target with a thickness $0.1\% L_{rad}$ at $\varepsilon = 25\ GeV$. The notations are the same as in Fig.1(a). For this thickness $T_c = 0.96$ and one has a thin target starting from $\omega \leq \omega_c$. So, we have here very wide plateau. The left edge of the plateau is defined by the contribution of transition radiation ($\omega \sim \omega_p$). Since in this case $4k = \nu_1^2 T = T_c \simeq 1$ (see (2.39)$\div$(2.42)), one has to calculate the ordinate of the plateau using the exact formula for a thin target (2.39). For this case the ratio of ordinate of the plateau to ordinate of the Bethe-Heitler intensity (see (3.8)) is $R \simeq 0.85$. The same ordinate has the plateau for electron energy $\varepsilon = 8\ GeV$ (Fig.2(b)). However, a width of the plateau for this electron energy is more narrow ($1\ MeV \div 20\ MeV$) due to diminishing of the interval between ω_p and ω_c. For $\omega > \omega_c$ the formation length of radiation becomes shorter than target thickness ($T = T_c\omega/\omega_c > 1$) and the parameter ν_1 decreases. The value $\nu_1 T = T_c\sqrt{\omega/\omega_c}$ increases with ω growth. A target becomes thick and the spectral curves is described by the Bethe-Heitler formula in Fig.2(b) starting from photon energy $\omega \sim 100\ MeV$. In Fig.2(b) the contributions of separate terms into Re $J^{(2)}$ are shown as well. Their behavior at $\omega > \omega_c$ is described quite satisfactory by formulae (2.29)-(2.30) (see also discussion at their derivation).

We compared our calculations with experimental data [7]. The curves T in Fig.1,2 give theory prediction (no fitting parameters !) in units $2\alpha/\pi$. We recalculated data according with given in [7] procedure

$$\left(\omega\frac{dw}{d\omega}\right)_{exp} = \frac{l}{L_{rad}}\frac{N_{exp}}{k}, \quad k = 0.09 \tag{4.1}$$

It is seen that in Fig.1(a) there is a perfect agreement of the theory and data. In Fig.1(b) there a overall difference: data is order of 10% are higher than theory curve. For photon energy $\omega = 500\ MeV$ the theory coincide with Bethe-Heitler formula (with the Coulomb corrections) applicable for this energy. Note that just for this case it was similar problem with normalization of data matching with the Migdal Monte Carlo simulation (+12.2%, see Table II in [7]).

For thickness $l = 0.1\%\ L_{rad}$ there is a qualitative difference between our theory prediction and Monte Carlo simulation in [7]. There was a number of experimental uncertainties associated with this target. Nevertheless, we show data for $\varepsilon = 25\ GeV$ which are lying higher than theory curve.

For target with a thickness $l = 0.7\% L_{rad}$ at $\varepsilon = 25\ GeV$ data was compared with calculation in [14] (the Coulomb corrections were discarded). After arbitrary diminishing of calculated value by 7% it was found excellent agreement. It is seen from the above analysis that this subtraction can be considered as taking account of the Coulomb corrections contribution.

We would like to thank S. Klein for useful comments about data.

References

[1] L. D. Landau and I. Ya. Pomeranchuk, Dokl.Akad.Nauk SSSR **92** (1953) 535, 735. See in English in *The Collected Paper of L. D. Landau*, Pergamon Press, 1965.

[2] A. B. Migdal, Phys. Rev. **103** (1956) 1811.

[3] A. B. Migdal, Sov. Phys. JETP **5** (1957) 527.

[4] M. L. Ter-Mikaelian, *High Energy Electromagnetic Processes in Condensed Media*, John Wiley & Sons, 1972.

[5] P. L. Anthony, R. Becker-Szendy, P. E. Bosted *et al*, Phys. Rev. Lett. **75** (1995) 1949.

[6] P. L. Anthony, R. Becker-Szendy, P. E. Bosted *et al*, Phys. Rev. Lett. **76** (1996) 3550.

[7] P. L. Anthony, R. Becker-Szendy, P. E. Bosted *et al*, Phys.Rev. **D56** (1997) 1373.

[8] V. N. Baier and V. M. Katkov, hep-ph 9709214, Preprint BINP 97-70, Novosibirsk, 1997, (to be published in Phys.Rev.D)

[9] I. S. Gradshtein and I. M. Ryzhik, *Table of Integrals, Series, and Products*, Academic Press, New York, 1965.

[10] X. Artru, G. B. Yodth and G. Mennesier Phys.Rev. **D12** (1975) 1289.

[11] F. F. Ternovskii Sov. Phys. JETP **12** (1960) 123.

[12] N. F. Shul'ga and S. P. Fomin JETP Lett. **63** (1996) 873.

[13] R.Blancenbeckler and S. D. Drell, Phys.Rev. **D53** (1996) 6265.

[14] B. G. Zakharov JETP Lett. **64** (1996) 781.

V.N. Baier and V.M. Katkov

The Landau-Pomeranchuk-Migdal effect in a thin target

Budker INP 97-105

Ответственный за выпуск А.М. Кудрявцев
Работа поступила 24.12. 1997 г.

Сдано в набор 25.12.1997 г.
Подписано в печать 25.12.1997 г.
Формат бумаги 60×90 1/16 Объем 1.8 печ.л., 1.5 уч.-изд.л.
Тираж 120 экз. Бесплатно. Заказ N° 105

Обработано на IBM PC и отпечатано на
ротапринте ИЯФ им. Г.И. Будкера СО РАН,
Новосибирск, 630090, пр. академика Лаврентьева, 11.