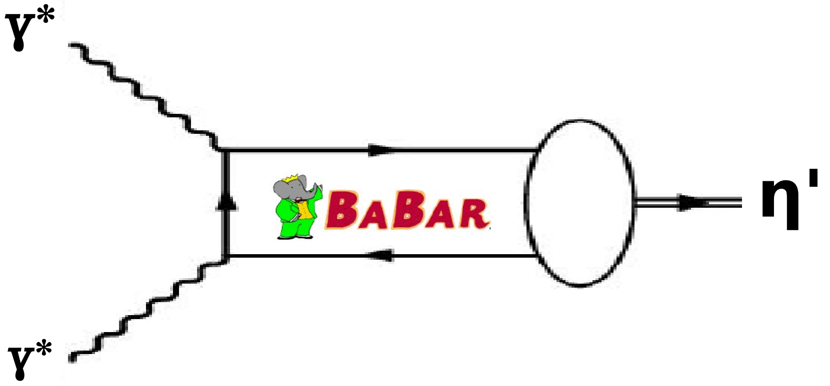
Experimental meeting, BINP, Novosibirsk

Pseudoscalar mesons transition form factors (TFF)
New study of $e^+e^- \to e^+e^-\eta'$ in the double-tag mode at BABAR

(based on arXiv:1808.08038)

Evgeny Kozyrev, Vladimir Druzhinin



$J^{PC} = 0^{-+}$

September 28, 2018

Outline

- Introduction

  -The definition of transition form factor (TFF)

  -Theoretical aspects

  -Existing experimental data

- Measurement of the TFF of η' meson with BaBar detector
- Comparison with theoretical predictions
- Summary
- Prospects for such investigations with VEPP-2000 and c/tau

M. Poppe, Int. J. Mod. Phys. A 1, 545 (1986):

At present, a major interest of $\gamma\gamma$ physics concerns the answer to the question "do the photons resolve the hadron's structure or not?" In other words: is particle production in $\gamma\gamma$ interactions primarily the production of quark pairs or is the VDM interpretation correct that the photons turn into vector mesons before they interact? In the latter case, two-photon physics would be just a continuation of fixed target hadron scattering experiments, and we would not expect great news to appear.

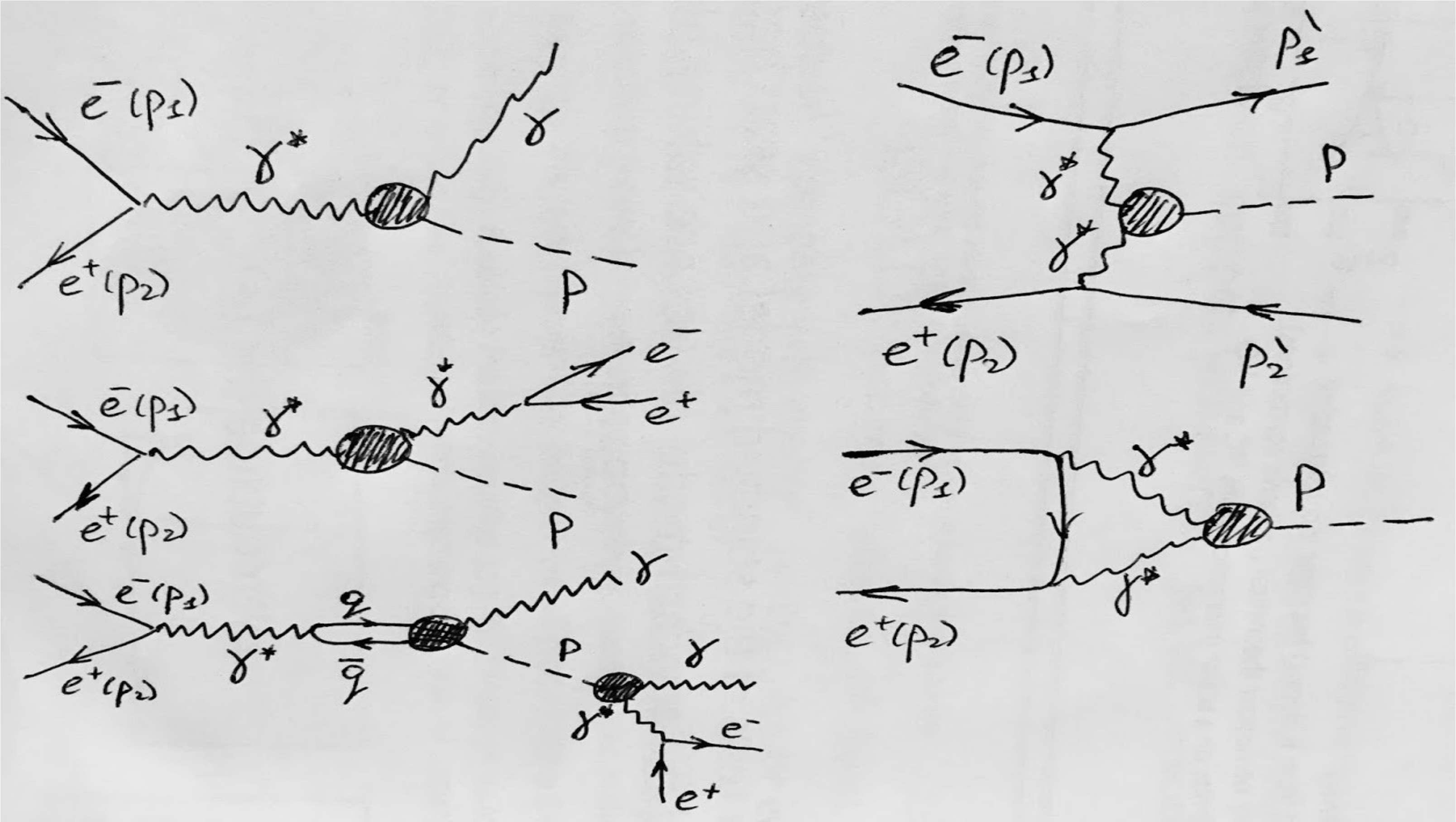
A.V. Radyushkin, R. Ruskov, Nuclear Physics B 481 (1996) 625-680:

	VMD	pQCD
$Q_1^2 \approx 0,\ Q_2^2 \to \infty$	$1/Q^2$	$1/Q^2$
$Q_1^2,\ Q_2^2 \to \infty$	$1/Q^4$	$1/Q^2$

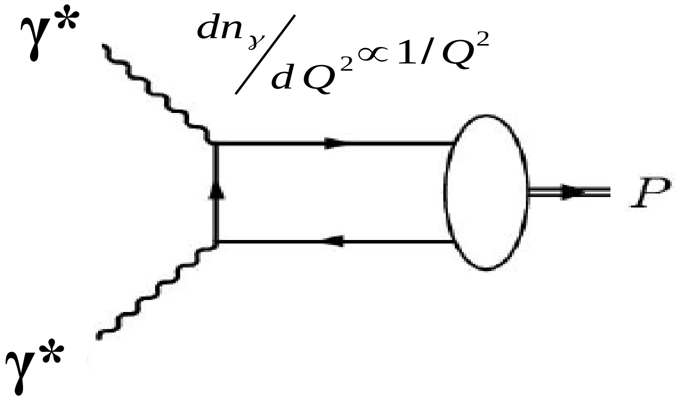
$$F^{LO}_{\gamma^*\gamma^*\pi^0}(q^2,Q^2) = \frac{4\pi}{3}\int_0^1 \frac{\varphi_\pi(x)}{xQ^2+\bar{x}q^2}\,dx,$$

where $\varphi_\pi(x)$ is the pion distribution amplitude and $x, \bar{x} \equiv 1-x$ are the fractions of the pion light-cone momentum carried by the quarks. In the region where both photon virtualities are large: $q^2 \sim Q^2 \gtrsim 1\ \mathrm{GeV}^2$, the pQCD predicts the overall $1/Q^2$ fall-off of the form factor, which differs from the naive vector meson dominance expectation $F_{\gamma^*\gamma^*\pi^0}(q^2,Q^2) \sim 1/q^2Q^2 \sim 1/Q^4$. Thus, establishing the $1/Q^2$ power law in this region is a crucial test of pQCD for this process. The study of $F_{\gamma^*\gamma^*\pi^0}(q^2,Q^2)$ over a wide range of the ratio q^2/Q^2 of two large photon virtualities can then provide non-trivial information about the shape of $\varphi_\pi(x)$.

Examples of experimental setups for the measurement of TFF



Дважды виртуальные фф легче мерить, меньше требований к качеству детектора, т. к. все частицы детектируются



P — pseudoscalar meson

$e_{1,2}$ — photon polarization

$q_{1,2}$ — 4-momentum of photon

$-Q^2 = q^2$

The amplitude of the γ*γ*→P transition:

$$A = e^2 \varepsilon_{\mu\nu\alpha\beta} e_1^\mu e_2^\nu q_1^\alpha q_2^\beta F(q_1^2, q_2^2),$$

• There are a lot of experimental study of pseudoscalar meson production via the fusion of real (**on-shell**) and virtual (**off-shell**) photons

γ*γ → P: π^0, η, η', η_c

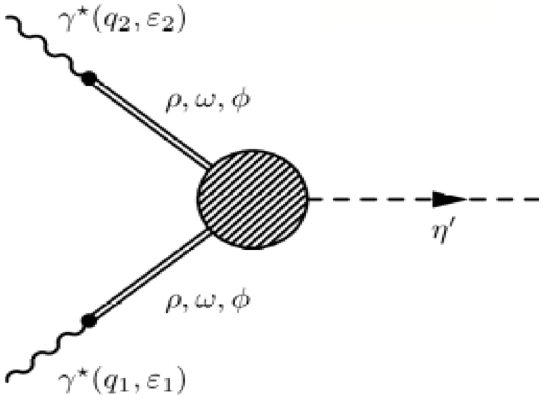
• There are **no** measurements of the double **off-shell** transitions

γ*γ* → P

- **η' deacay to real photons:**

$$\Gamma_{\eta' \to 2\gamma} = \frac{\pi \alpha^2 m_{\eta'}^3}{4} |F(0,0)|^2 = 4.30 \pm 0.16 \text{ keV} \quad \Rightarrow \quad F(0,0) = 0.342 \pm 0.006 \text{ GeV}^{-1}.$$

- The vector meson dominance model is commonly used to describe TFF at low Q^2:



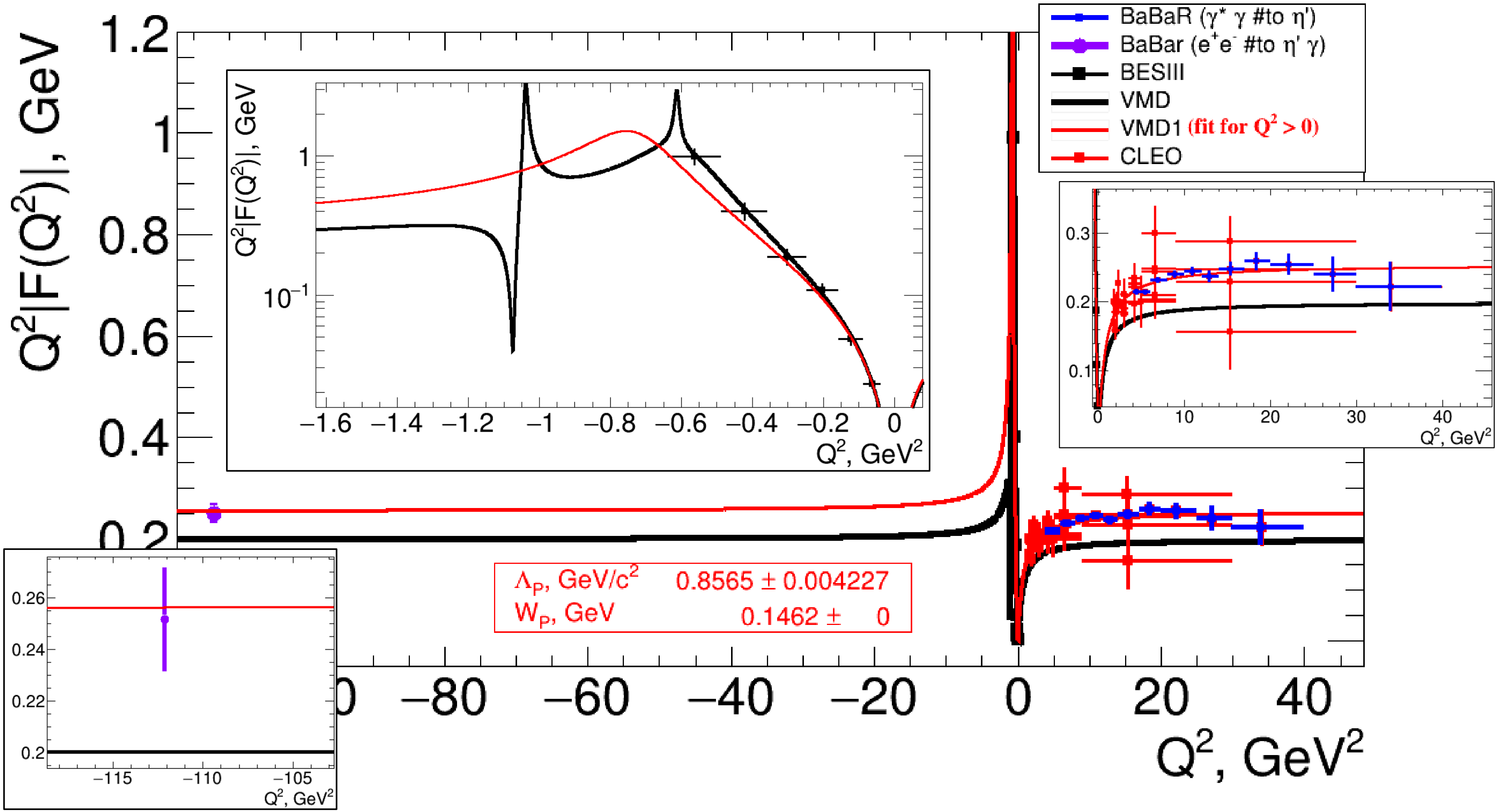
$$F(Q^2) = \frac{\sum_V \frac{f_{PV\gamma}}{f_V} \cdot \frac{m_V^2}{m_V^2 - q^2 - i W_V m_V} \cdot F(0)}{\sum_V \frac{f_{PV\gamma}}{f_V}}$$

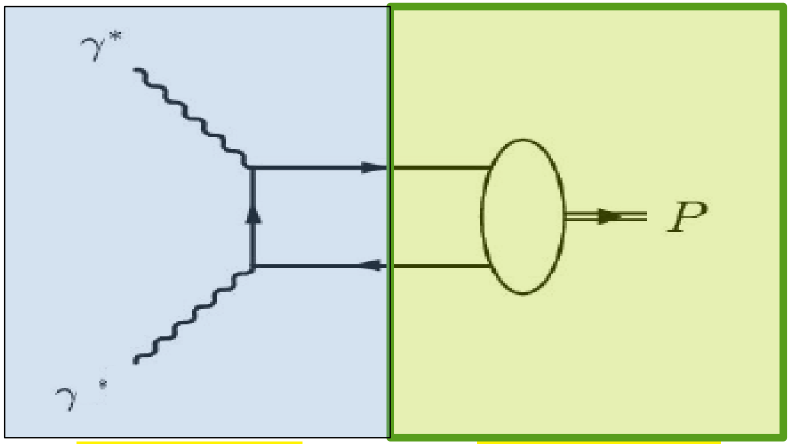
PDG
$W(\eta` \to \rho\gamma)$
$W(\eta` \to \omega\gamma)$
$W(\varphi \to \eta`\gamma)$
$W(\varphi \to ee)$
$W(\rho \to ee)$
$W(\omega \to ee)$

$$B(\phi \to \eta'\gamma) = (11,4^{+5,4}_{-4,4} \pm 2,0) \cdot 10^{-5}.$$

- In double off-shell case at $Q^2 > W_V m_V$: $F_{\eta'}(Q_1^2, Q_2^2) = \dfrac{F_{\eta'}(0,0)}{(1 + Q_1^2/\Lambda_P^2)(1 + Q_2^2/\Lambda_P^2)}$

  where Λ_P — effective pole mass parameter


BaBaR (γ* γ #to η')
BaBar (e+e- #to η' γ)
BESIII
VMD
VMD1 (fit for Q² > 0)
CLEO
Λ_P, GeV/c² 0.8565 ± 0.004227
W_P, GeV 0.1462 ± 0
Q²|F(Q²)|, GeV
Q², GeV²



$$F(Q^2{}_1,Q^2{}_2) = \int \boxed{T(x,Q^2{}_1,Q^2{}_2)}\ \boxed{\varphi(x,Q^2{}_1,Q^2{}_2)}\ dx$$

x - is the fraction of the meson momentum carried by one of the quarks

$T(x,Q^2{}_1,Q^2{}_2)$ - hard scattering amplitude for $\gamma^*\gamma^*\to$qqbar transition which is calculable in pQCD

$\varphi(x,Q^2{}_1,Q^2{}_2)$ - nonperturbative meson distribution amplitude (DA) describing transition P → qqbar

$$T_H(x,Q_1^2,Q_2^2) = \frac{1}{2}\cdot\frac{1}{xQ_1^2+(1-x)Q_2^2}\cdot\left(1+C_F\frac{\alpha_S(Q^2)}{2\pi}\cdot t(x,Q_1^2,Q_2^2)\right)+(x\to 1-x)+O(\alpha_s^2)+O(\Lambda_{QCD}^4/Q^4)$$

NLO correction [E. Braaten, Phys. Rev. D **28**, 3 (1983)]

- The shape (x dependence) of meson DA **$\varphi(x,Q^2{}_1,Q^2{}_2)$** is unknown, but its evolution with **$\mu^2 = Q^2{}_1+Q^2{}_2$** is predicted by pQCD:

$$\mu^2\frac{d}{\mu^2}\phi(x,\mu) = \frac{\alpha_s(\mu)}{2\pi}\int_0^1 dyV(x,y)\phi(y,\mu)$$

At the limit **$\mu\to\infty$**

$$\phi_P(x,\mu) = A_P 6x(1-x)(1+O(\Lambda_{QCD}^2/\mu^2))$$

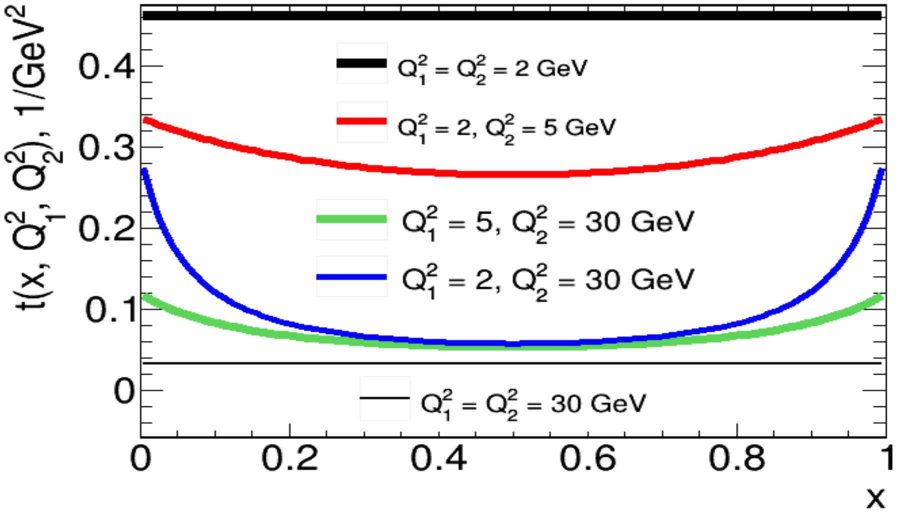
[S. J. Brodsky and G. P. Lepage, Phys. Rev. D 24, 7 (1981)]

$$F_{\eta'}(Q_1^2,Q_2^2) = \left(\frac{5\sqrt{2}}{9} f_n \sin\phi + \frac{2}{9} f_s \cos\phi\right) \int_0^1 dx \frac{1}{2} \frac{\boxed{6x(1-x)}}{xQ_1^2 + (1-x)Q_2^2} \left(1 + C_F \frac{\alpha_s(\mu^2)}{2\pi} \cdot t(x,Q_1^2,Q_2^2)\right) + (x \to 1-x),$$

Master formula

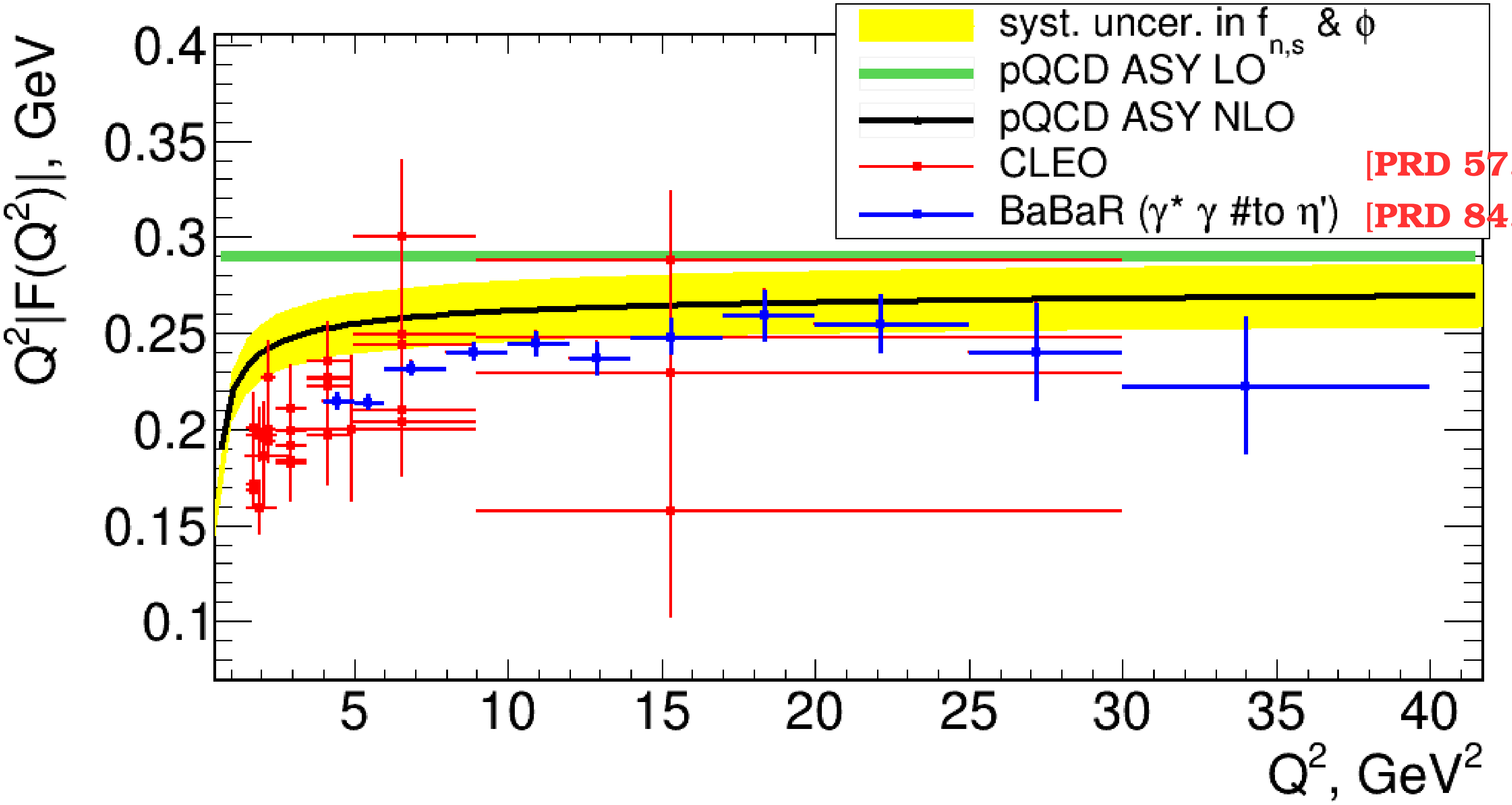
The meson DA

NLO



- The double-virtual TFF is **less sensitive to NLO** than the single off-shell TFF.

- The form $\boxed{1/[xQ_1^2+(1-x)Q_2^2]}$ is not divergent, so double off-shell transition FF is **less sensitive to a shape of the meson DA** in comparison to the single off-shell FF.



[PRD 57, 33 (1998)]

[PRD 84, 052001]

The $\gamma^*\gamma \to \eta$ Transition Form Factor

Additionally, the study is motivated by g-2 puzzle

Pseudoscalar pole contribution to the hadronic light-by-light piece of aμ

Adolfo Guevara, Pablo Roig, JJ Sanz Cillero. Sep 17, 2018. 7 pp.

Conference: C18-06-25.2

e-Print: arXiv:1809.06175

is the largest one. A way to reduce such uncertainty could be by taking into account data form doubly off-shell TFF such as that given by BaBar for the η'-TFF [35]. Considering all possible contributions to the error we get

$$a_\mu^{P,HLbL} = \left(8.47 \pm 0.16_{\text{sta}} \pm 0.09_{1/N_C}\,{}^{+0.5}_{-0}{}_{\text{asym}}\right) \cdot 10^{-10}, \quad (14)$$

where the first error (sta) comes from the fit of the TFF, the second from possible $1/N_C$ corrections and the last from the wrong asymptotic behavior estimated through the effects of heavier resonances in the TFF.

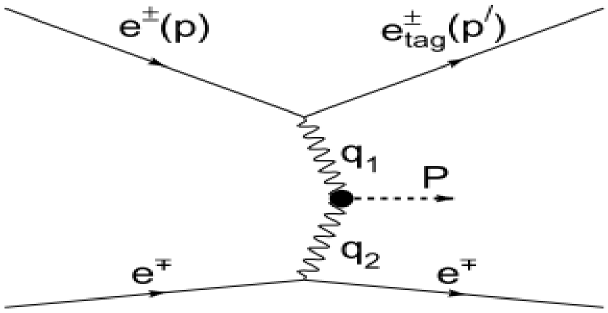
The analysis is based on the previous BaBar study [1].

Previous

$\gamma\gamma^* \to \eta'$

Single tagged

~ 5000 signal events

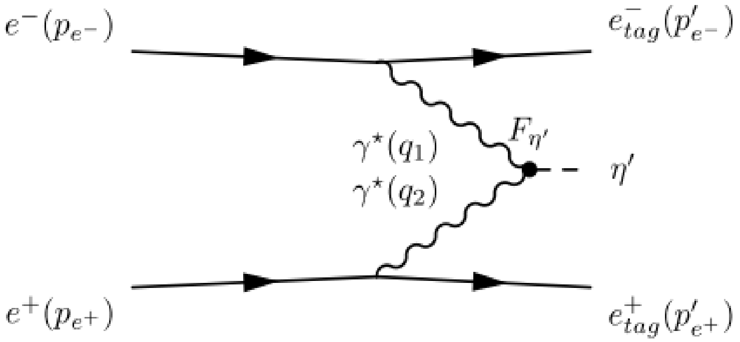


New

$\gamma^*\gamma^* \to \eta'$

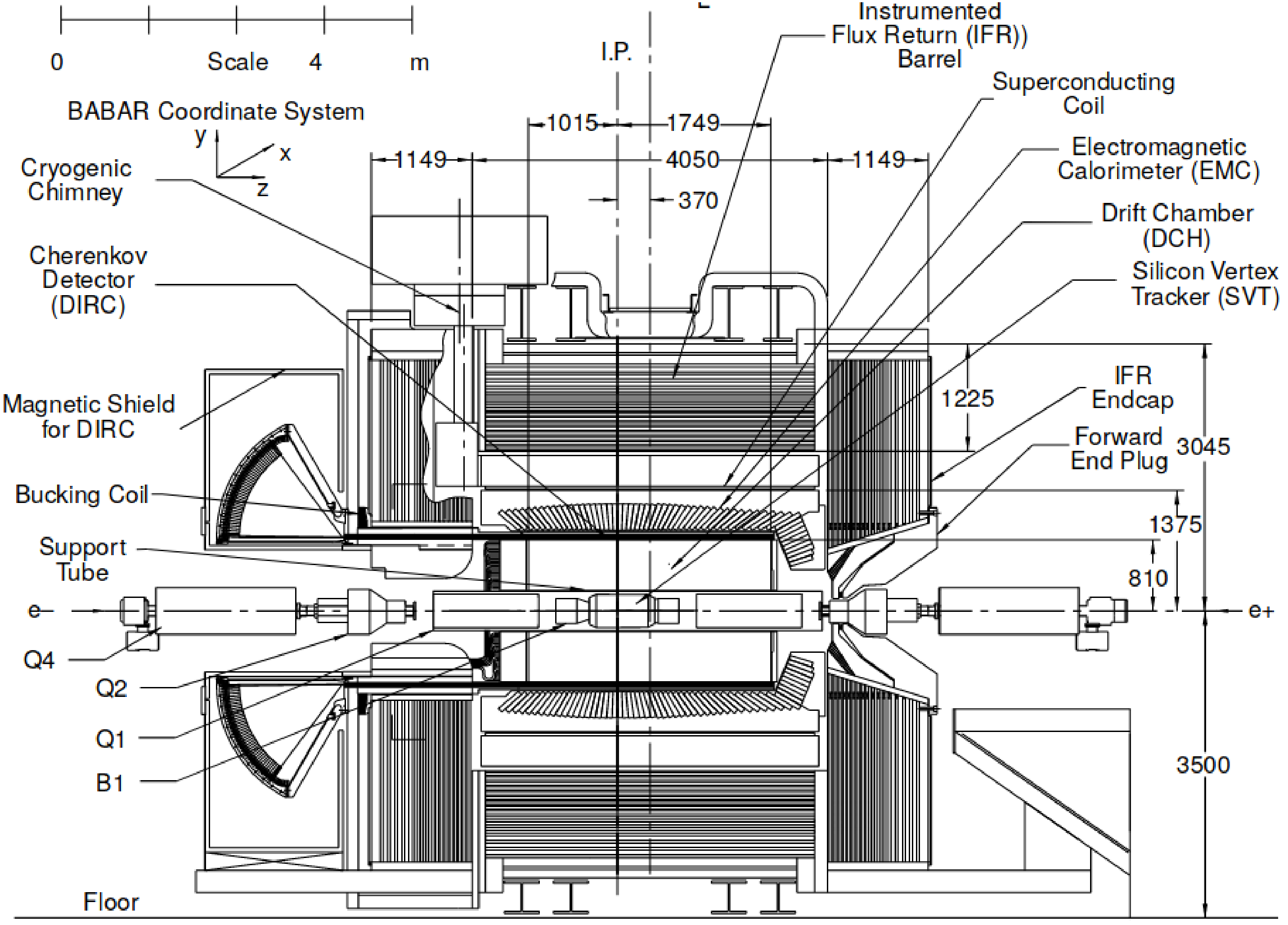
Double tagged

46^{+8}_{-7} signal events

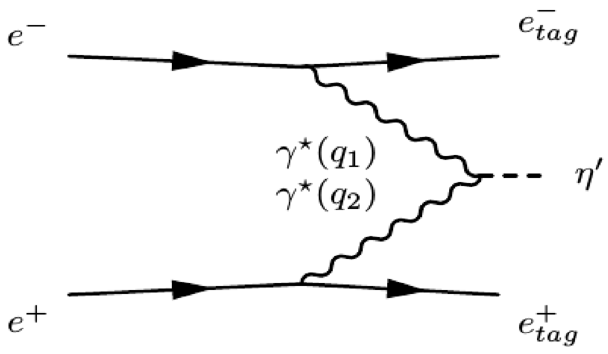


- **A large number of systematic uncertainties were studied in our previous work where the number of signal events was significantly larger.**

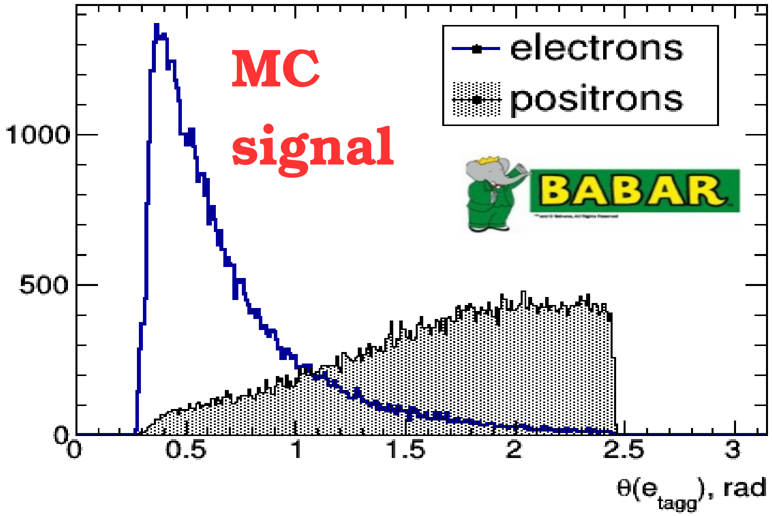
[1] [PRD 84, 052001]: P. del Amo Sanchez *et al. (BaBar collaboration)*, Phys. Rev. D 84, 052001 (2011) — (126 citations).


0
Scale
4
m
BABAR Coordinate System
y
x
z
I.P.
Instrumented
Flux Return (IFR))
Barrel
Superconducting
Coil
Electromagnetic
Calorimeter (EMC)
Drift Chamber
(DCH)
Silicon Vertex
Tracker (SVT)
IFR
Endcap
Forward
End Plug
1015
1749
1149
4050
1149
370
1225
3045
1375
810
3500
Cryogenic
Chimney
Cherenkov
Detector
(DIRC)
Magnetic Shield
for DIRC
Bucking Coil
Support
Tube
e−
e+
Q4
Q2
Q1
B1
Floor

$$q^2_{e^-} = -Q^2_{e^-} = (p_{e^-} - p'_{e^-})^2$$



$$q^2_{e^+} = -Q^2_{e^+} = (p_{e^+} - p'_{e^+})^2$$



Polar angle distribution for tagged electrons (positrons)

- The decay chain $\eta' \rightarrow \pi^+\pi^-\eta \rightarrow \boldsymbol{\pi^+\pi^-2\gamma}$ is used
- A total integrated luminosity L = 469 fb^{-1}
- GGResRc event generator is used [arXiv:1010.5969]. Initial and final state radiative corrections as well as vacuum polarization effects are included. The form factor is fixed to the constant value F(0,0).

The strategy: $dN/dQ^2 \Rightarrow d\sigma/dQ^2 \Rightarrow |F(Q^2)|$

We require the presense

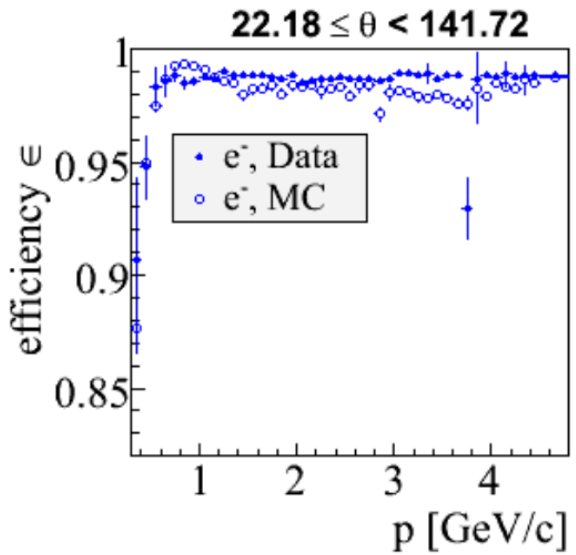
- at least **two tracks** from *GoodTrackLoose* list passed *LooseElectronMicroSecection*
- at least **two tracks** from *GoodTrackLoose* list passed *TightKMPionMicroSelection*
- at least **two photons** from *GoodPhotonLoose* list

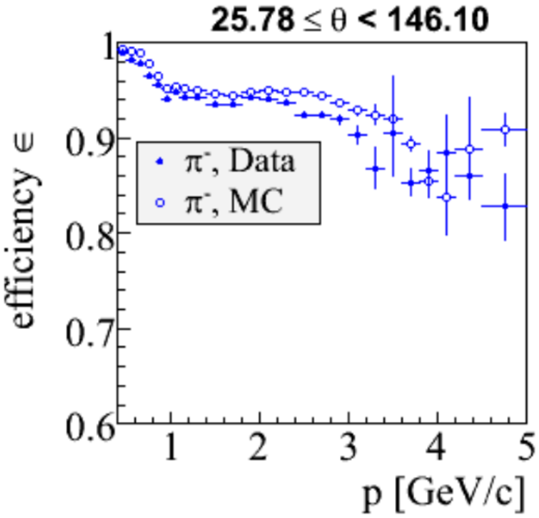
 -ε_γ > 30 MeV

 -0.45 < $m_{\gamma\gamma}$ <0.65 GeV/c^2

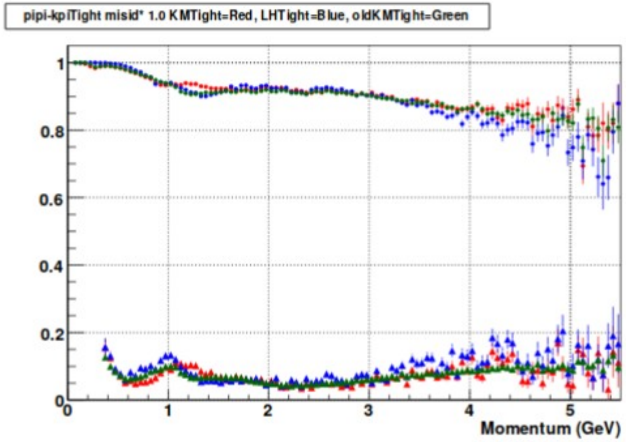
 -The photon candidates are fitted with a η mass constraint.

- The η candidate and a pair of oppositely-charged pion candidates are fitted with a η' mass constraint.

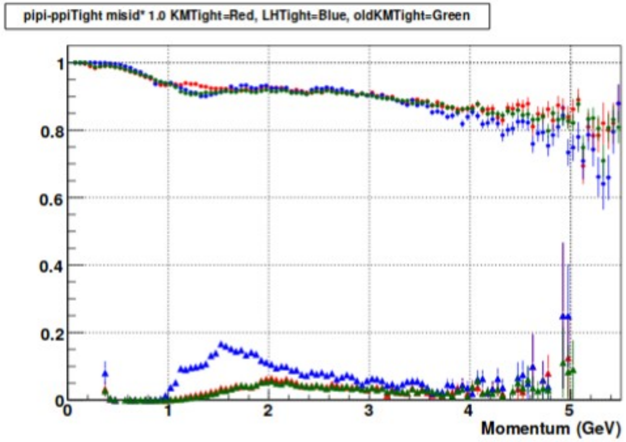




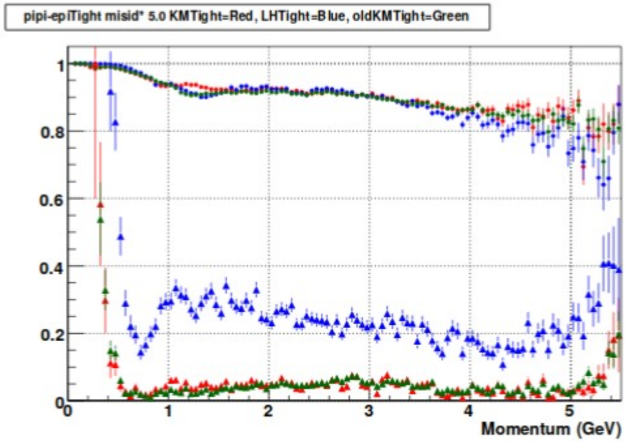
Pions misedentification with *TightKMPionMicroSelection:*



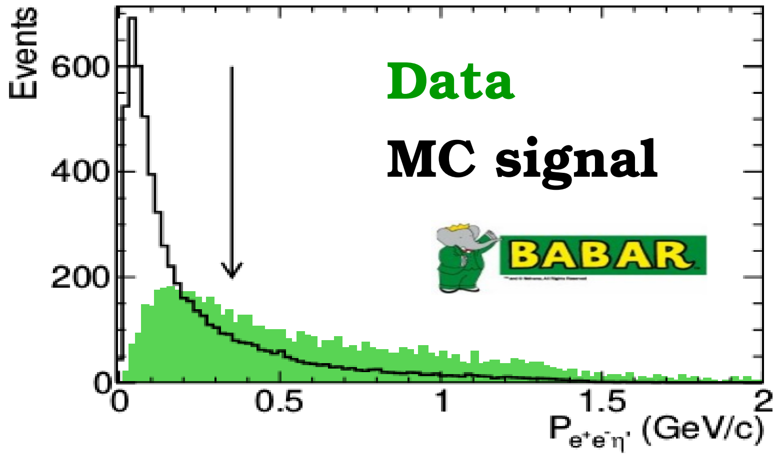
(a) K as π mis-id*1



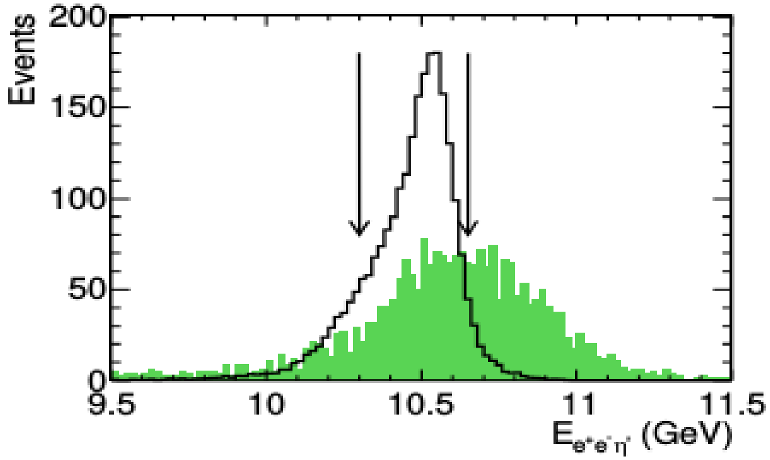
(b) p as π mis-id*1



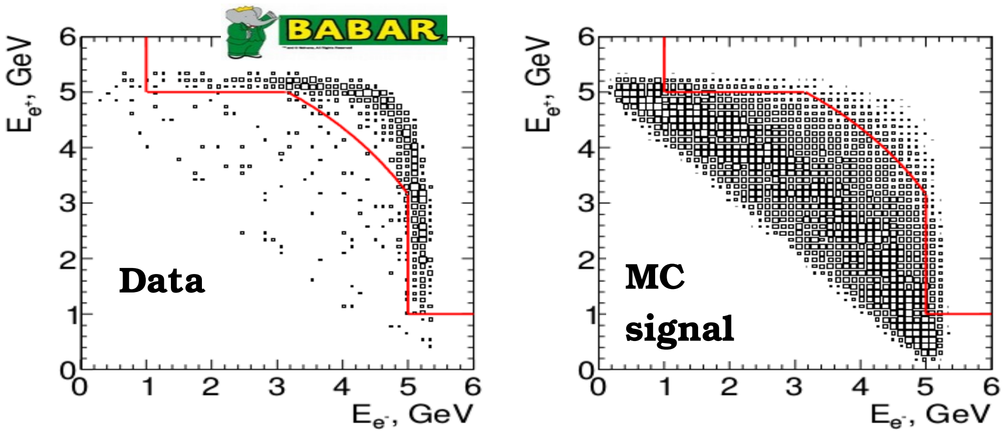
- $P_{c.m.}(e^+e^-\pi^+\pi^-\eta) < 0.35$ GeV/c.



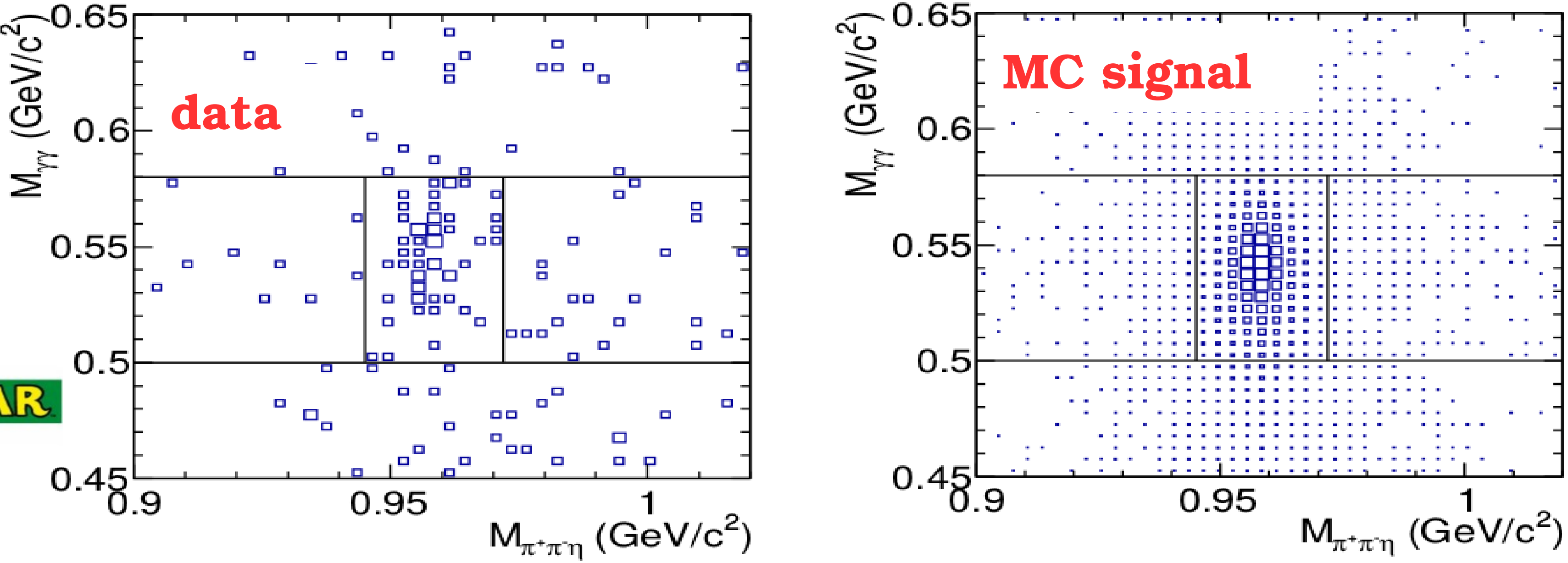
- $10.3 < E_{c.m.}(e^+e^-\pi^+\pi^-\eta) < 10.7$ GeV



- Events that lie above and on the right of the lines (mostly, Bhabha scattering) are rejected.

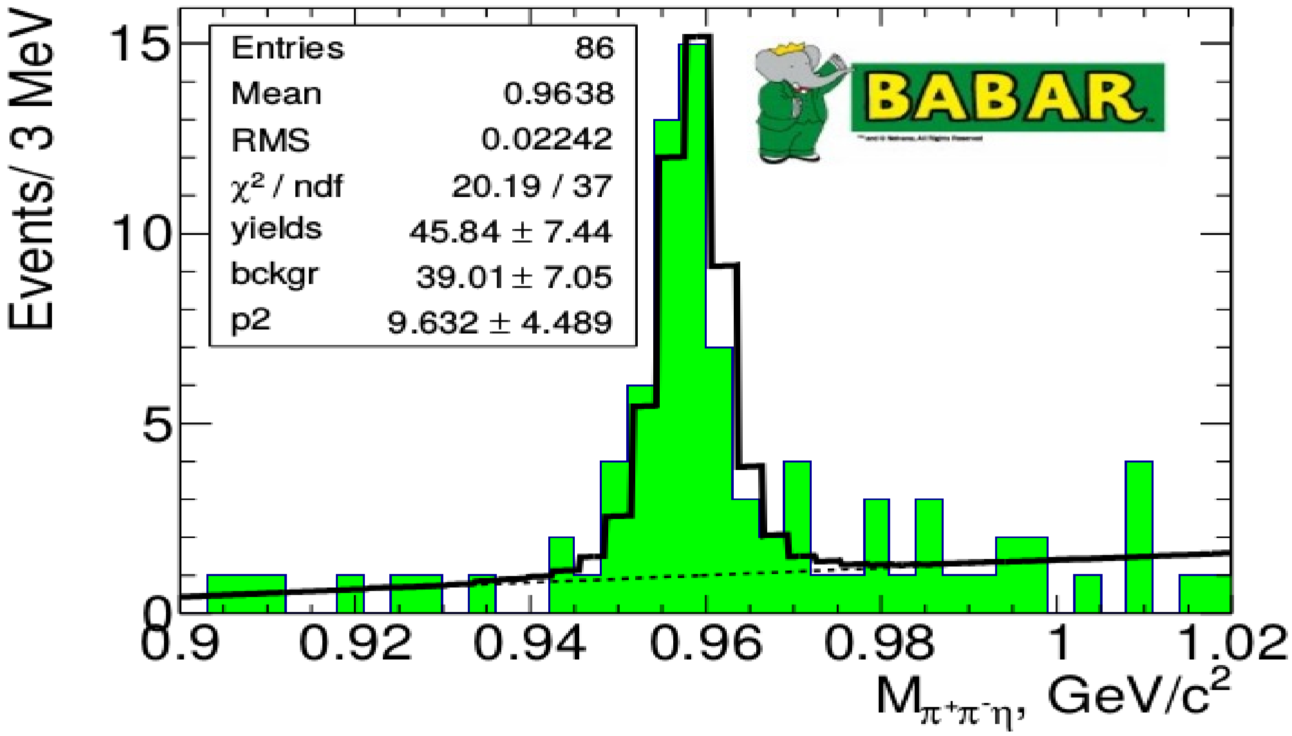


The positron c.m. energy vs the electron c.m. energy

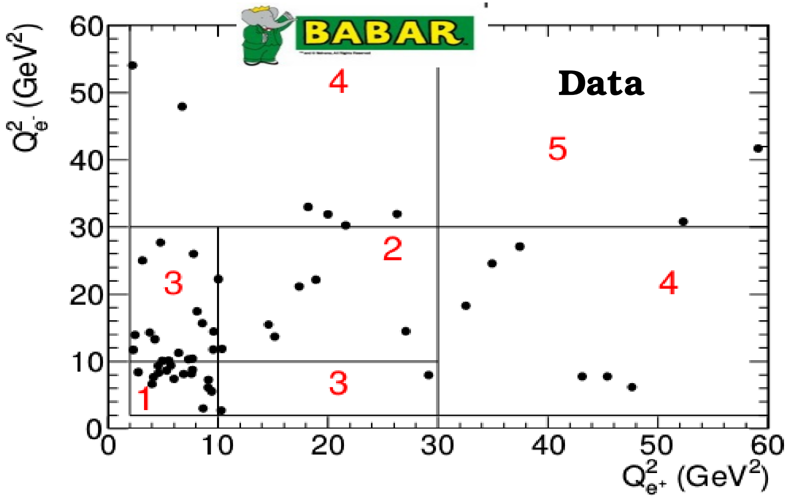


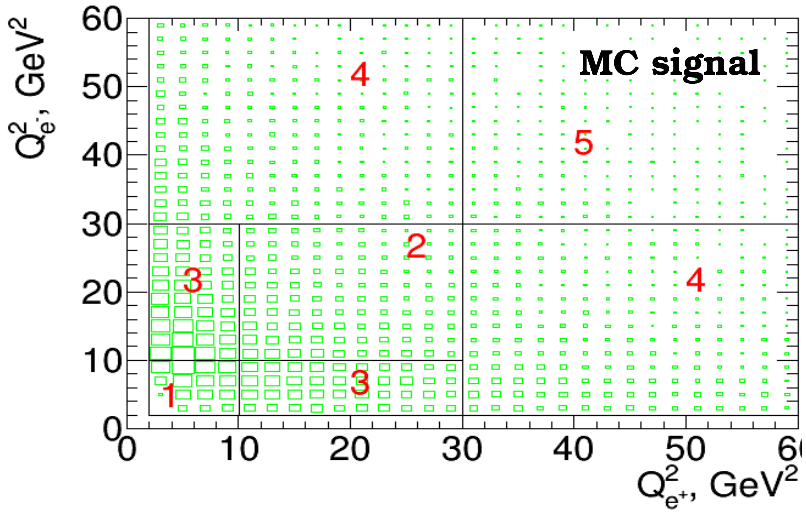
$$m_{\gamma\gamma}\ vs.\ m_{\pi+\pi-\eta}$$

- We require $0.50 < m_{\gamma\gamma} < 0.58$ GeV/c^2



The $\pi^+\pi^-\eta$ mass spectra for data events. The open histogram is the fit result. The dashed line represents fitted background.





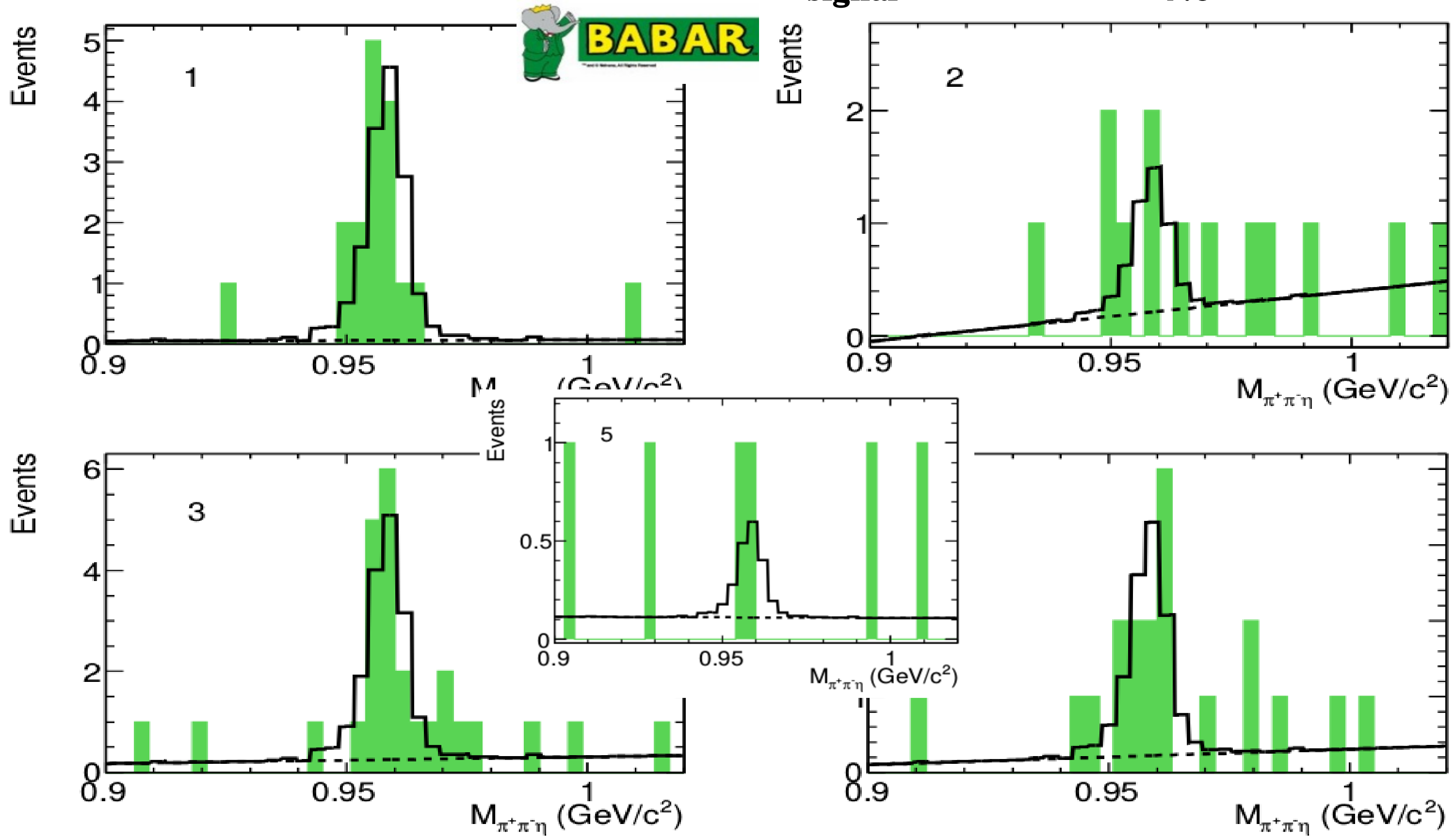
The Q^2_{e-} vs. Q^2_{e+} for events with $\mathbf{0.945 < m_{2\pi\eta} < 0.972}$ *GeV/c²*

- New definition: $Q_1^2 = \max(Q^2_{e+}, Q^2_{e-}),\ Q_2^2 = \min(Q^2_{e+}, Q^2_{e-})$

- The average momentum transfers for each region are calculated using the data spectrum normalized to the detection efficiency:

$$\overline{Q^2_{1,2}} = \frac{\sum_i Q^2_{1,2}(i)/\varepsilon(Q_1^2, Q_2^2)}{\sum_i 1/\varepsilon(Q_1^2, Q_2^2)}$$

- The total number of signal events $\mathbf{N^{fit}_{signal} = 46.2^{+8.3}_{-7.0}}$

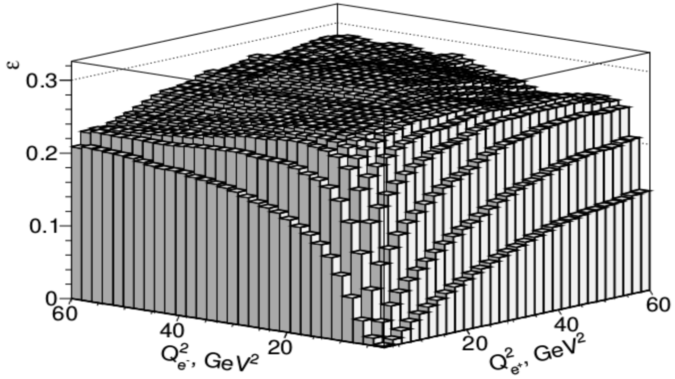


The $\pi^+\pi^-\eta$ mass spectra for data events for the five Q^2 ranges. The open histograms are the fit results. The dashed lines represent background.

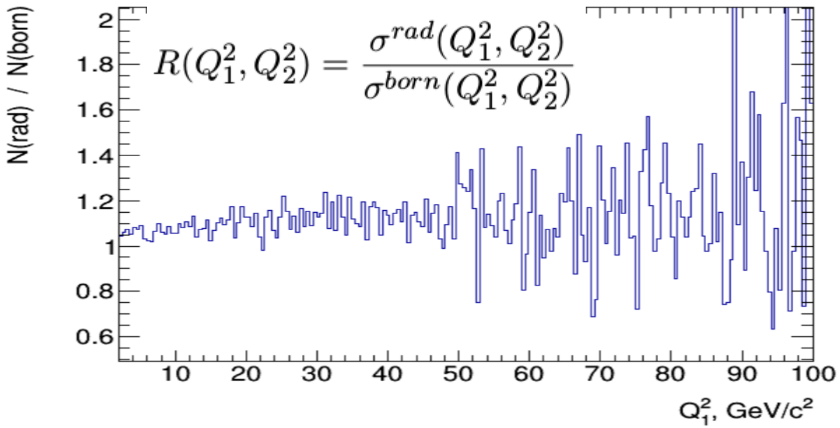
- The detector acceptance limits the e^-e^+ detection efficiency at small Q^2 . The minimum Q^2 equals to 2 GeV^2.

$$\boxed{\varepsilon_{true} = \frac{\int \varepsilon(Q_1^2, Q_2^2) F^2_{\eta'}(Q_1^2, Q_2^2) dQ_1^2 dQ_2^2}{\int F^2_{\eta'}(Q_1^2, Q_2^2) dQ_1^2 dQ_2^2}}$$

($\boldsymbol{F}_{\eta'}$ from *master formula* at slide #7)



The dependence of detection efficiency on momentum transfers.



The ratio of generated spectra with rad. photons vs. without photons

- R leads to the decrease of the detection efficiency by ~10 %.

- The maximum energy of the photon emitted from the initial state is restricted by the requirement $E_\gamma < 0.05\sqrt{s}$, where $\sqrt{s}$ is the e^+e^- center-of-mass (c.m.) energy.

Cross section and Form Factor

- The differential cross section for $e^+e^- \to e^+e^-\eta'$ is calculated as

$$\boxed{\frac{d^2\sigma}{dQ_1^2 dQ_2^2} = \frac{1}{\varepsilon_{\rm true} R L B}\frac{d^2 N}{dQ_1^2 dQ_2^2}}$$

$$F^2(\overline{Q_1^2}, \overline{Q_2^2}) = \frac{(d^2\sigma/(dQ_1^2 dQ_2^2))_{data}}{(d^2\sigma/(dQ_1^2 dQ_2^2))_{MC}} F^2_{\eta'}(\overline{Q_1^2}, \overline{Q_2^2})$$

- $B = B(\eta' \to \pi^+\pi^-\eta) \times B(\eta \to 2\gamma) = (0.3941 \pm 0.0020) \times (0.429 \pm 0.007) = 0.169 \pm 0.003$
- $\boldsymbol{\sigma_{e+e- \to e+e-\eta'} (2 < Q_1^2, Q_2^2 < 60\ \mathrm{GeV}^2) = (11.4^{+2.8}_{-2.4})\ \mathrm{fb}}$

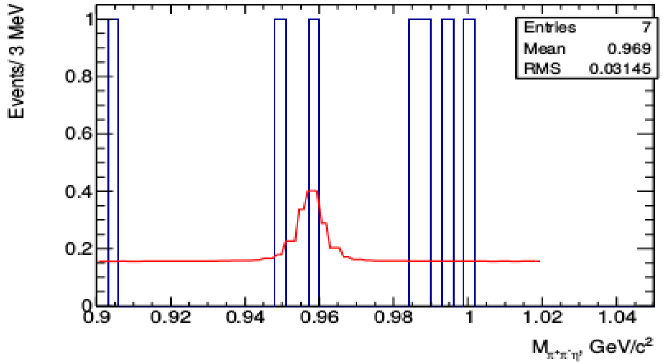
$\overline{Q_1^2}$, $\overline{Q_2^2}$, GeV^2	$\varepsilon_{\rm true}$	R	$N_{\rm events}$	$d^2\sigma/(dQ_1^2 dQ_2^2)$ $\times 10^4$, fb/GeV^4	$F(\overline{Q_1^2}, \overline{Q_2^2})$ $\times 10^3$, GeV^{-1}
6.48, 6.48	0.019	1.03	$14.7^{+4.3}_{-3.6}$	$1471.8^{+430.1}_{-362.9}$	$14.32^{+1.95}_{-1.89} \pm 0.83 \pm 0.14$
16.85, 16.85	0.282	1.10	$4.1^{+2.7}_{-2.7}$	$4.2^{+2.8}_{-2.8}$	$5.35^{+1.54}_{-1.54} \pm 0.31 \pm 0.42$
14.83, 4.27	0.145	1.07	$15.8^{+4.8}_{-4.0}$	$39.7^{+12.0}_{-10.2}$	$8.24^{+1.16}_{-1.13} \pm 0.48 \pm 0.65$
38.11, 14.95	0.226	1.11	$10.0^{+3.9}_{-3.2}$	$3.0^{+1.2}_{-1.0}$	$6.07^{+1.09}_{-1.07} \pm 0.35 \pm 1.21$
45.63, 45.63	0.293	1.22	$1.6^{+1.8}_{-1.1}$	$0.6^{+0.7}_{-0.6}$	$8.71^{+3.96}_{-8.71} \pm 0.50 \pm 1.04$

Statistical **Systematic** **Model**

- The statistical uncertainty is dominant

Systematic uncertainty. Background subtraction.

- **$e^+e^- \to e^+e^-\eta'\pi^0 \to e^+e^-\pi^-\pi^+\eta\pi^0$** - kinematically closest background for the process under study. Using the simulation of the $e^+e^- \to e^+e^- a_0(1450) \to e^+e^-\eta'\pi^0$ process we estimate the contribution $N_{\eta'\pi 0} < 0.16$ at 90% C.L.



The $\pi^+\pi^-\eta$ invariant mass spectrum

$$N^{signal}_{\eta'\pi^0} < 1.45 \text{ at } 90\% \text{ C.L.}$$

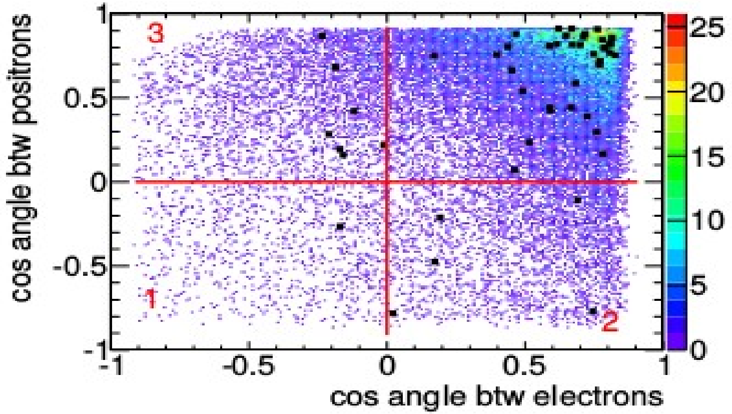
The detection efficiency for $e^+e^-\eta'\pi^0$ events to pass the selections of **$e^+e^-\eta'$**.

$$\mathrm{N}_{bkgr} = \frac{N^{signal}_{\eta'\pi^0}\,\epsilon^{(2)}_{\eta'\pi^0}}{\epsilon^{(1)}_{\eta'\pi^0}} < 0.16 \text{ at } 90\% \text{ C.L.}$$

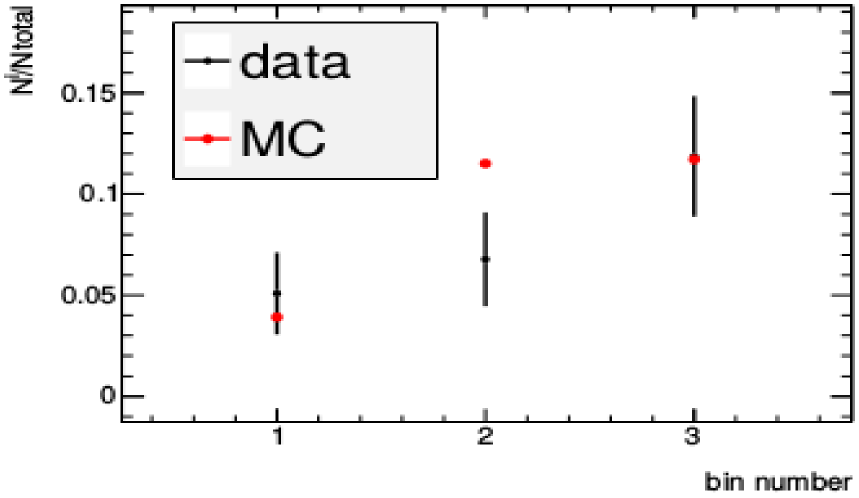
The detection efficiency for $e^+e^-\eta'\pi^0$ events to pass the selections of **$e^+e^-\eta'\pi^0$**.

Systematic uncertainty. Background subtraction.

- $e^+e^- \to e^+e^- J/\psi(\varphi) \to e^+e^-\eta'\gamma$ is negligible according to [**PRD 84, 052001**].
- $e^+e^- \to \gamma^* \to X$:



The cosine of angle between scattered and initial electron (positron) in c.m.f.



The fraction of the events in the bins.

It is reasonable to assume that the $\cos(\alpha_{e\pm})$ spectrums must be symmetric in [-1:1] region for **annihilation processes**, while signal scattered electron (positron) prefers to fly in the about the same direction.

The main source of systematic uncertainty of cross section

Source	Uncertainty (%)
$\pi^{\pm}$ identification	1.0
$e^{\pm}$ identification	1.0
Other selection criteria	11
Track reconstruction	0.9
$\eta \to 2\gamma$ reconstruction	2
Trigger, filters	1.3
Background subtraction	3.7
Radiative correction	1.0
Luminosity	1.0
Total	12%

from previous BaBar study of γ*γ→η′ **[PRD 84, 052001]**

selection	$N_{signal}/\varepsilon_{true}$	deviation from standard criteria
standard selection criteria	985 ± 197	
$P_{e^+e^-\eta'}$ is less than 1 GeV/c instead of 0.35 GeV/c	1052 ± 273	6.8
$10.20 < E_{e^+e^-\eta'} < 10.75$ GeV instead of $10.3 < E_{e^+e^-\eta'} < 10.65$ GeV	942 ± 235	-4.3
without the restrictions on E_{e^+} and E_{e^-}	1061 ± 280	7.7
$0.48 < m_{2\gamma} < 0.60$ GeV/c^2 instead of $0.50 < m_{2\gamma} < 0.58$ GeV/c^2	958 ± 181	-2.7
total		11

Model uncertainty

- **$(d^2\sigma/(dQ^2_1\ dQ^2_2))_{MC}$** and **$\varepsilon_{true}$ depends on model**.

  Repeating the calculations with a constant TFF we estimate the model uncertainty.

  For the cross section - about 60% due to the strong dependence of ε_{true} on the input model for TFF at small values of Q^2_1 and Q^2_2.

  The TFF is much less sensitive to the model.

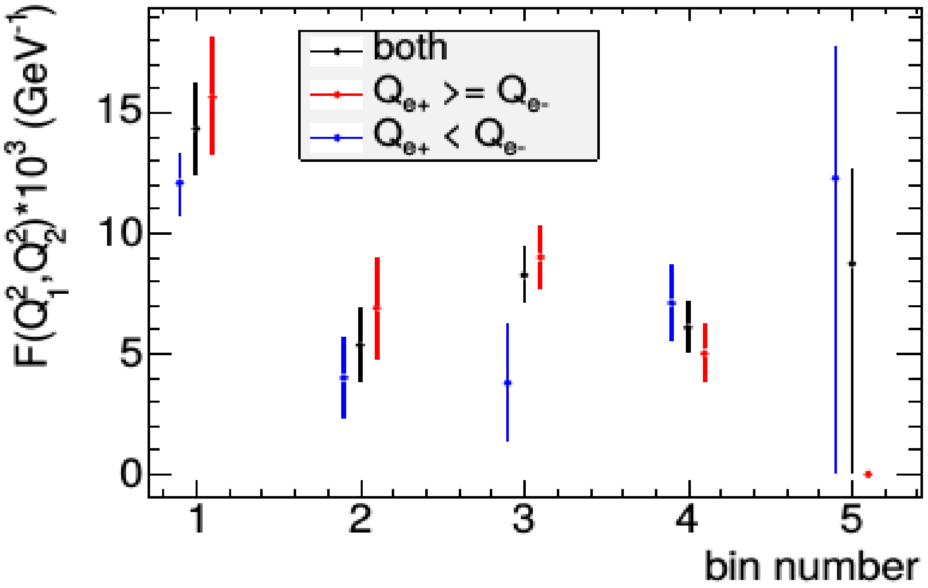
TABLE V: $\frac{d^2\sigma}{dQ_1^2 dQ_2^2}$ obtained with different models for TFF

	1	2	3	4	5
QCD	$1471.8^{+430.136}_{-362.91}$	$4.17^{+2.75}_{2.75}$	$39.72^{+11.98}_{-10.18}$	$2.98^{+1.17}_{-0.96}$	$0.62^{+0.69}_{-0.62}$
const	$637.10^{+186.19}_{-157.09}$	$4.15^{+2.74}_{2.74}$	$33.30^{+10.05}_{-8.54}$	$2.76^{+1.08}_{-0.89}$	$0.62^{+0.69}_{-0.62}$
deviation, %	60	0.6	15	7	1.

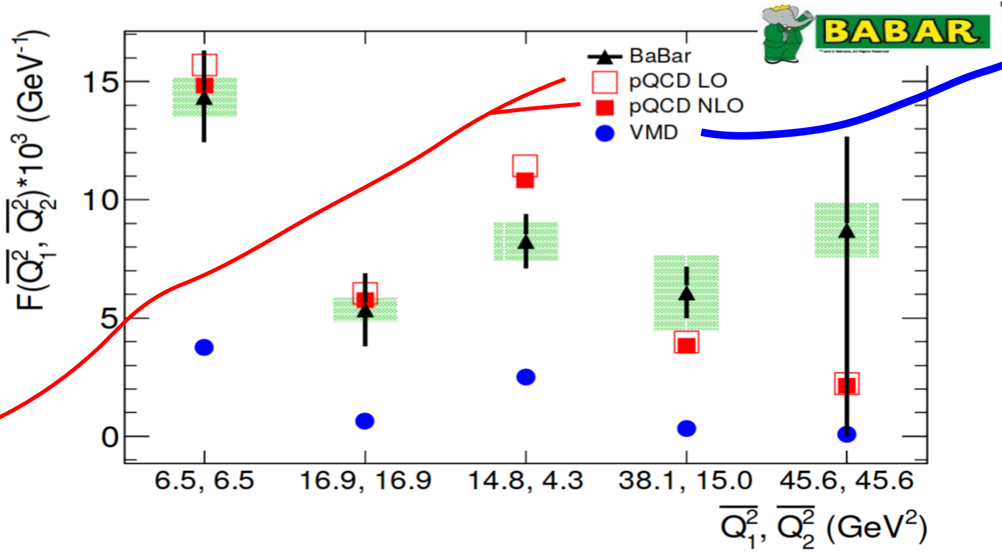
TABLE VI: TFF obtained with different models for TFF

	1	2	3	4	5
QCD	$14.32^{+1.95}_{-1.89}$	$5.35^{+1.54}_{-1.54}$	$8.24^{+1.16}_{-1.13}$	$6.07^{+1.09}_{-1.07}$	$8.71^{+3.96}_{-8.71}$
const	$14.61^{+1.99}_{-1.92}$	$5.62^{+1.62}_{-1.62}$	$7.24^{+1.02}_{-0.99}$	$7.24^{+1.30}_{-1.28}$	$10.02^{+4.55}_{-10.02}$
deviation %	1	8	8	20	12

Cross check



The comparison of the measured η′ TFF with
Q^2_{e+} < Q^2_{e-}, Q^2_{e+} >= Q^2_{e-} and without the restriction.



The comparison of obtained form-factor with theoretical predictions. Error bars - statistical uncertainties. Shaded rectangles - quadratic sum of the systematic and model uncertainties.

$$F_{\eta'}(Q_1^2, Q_2^2) = \frac{F_{\eta'}(0,0)}{(1+Q_1^2/\Lambda_P^2)(1+Q_2^2/\Lambda_P^2)}$$

The Λ_P is fixed at 849 MeV/c² from the approximation of $F_{\eta'}(Q^2, 0)$ with one off-shell photon [Phys. Rev. D 85, 057501 (2012)].

NLO

$$F_{\eta'}(Q_1^2, Q_2^2) = \left(\frac{5\sqrt{2}}{9} f_n \sin\phi + \frac{2}{9} f_s \cos\phi\right) \int_0^1 dx \frac{1}{2} \frac{6x(1-x)}{xQ_1^2 + (1-x)Q_2^2} \left(1 + C_F \frac{\alpha_s(\mu^2)}{2\pi} \cdot t(x, Q_1^2, Q_2^2)\right) + (x \to 1-x),$$

- pQCD calculation is in good agreement with data (χ^2/n.d.f. = 6.2/5, Prob = 28%)
- VMD model exhibits a clear disagreement with the experiment.

Summary

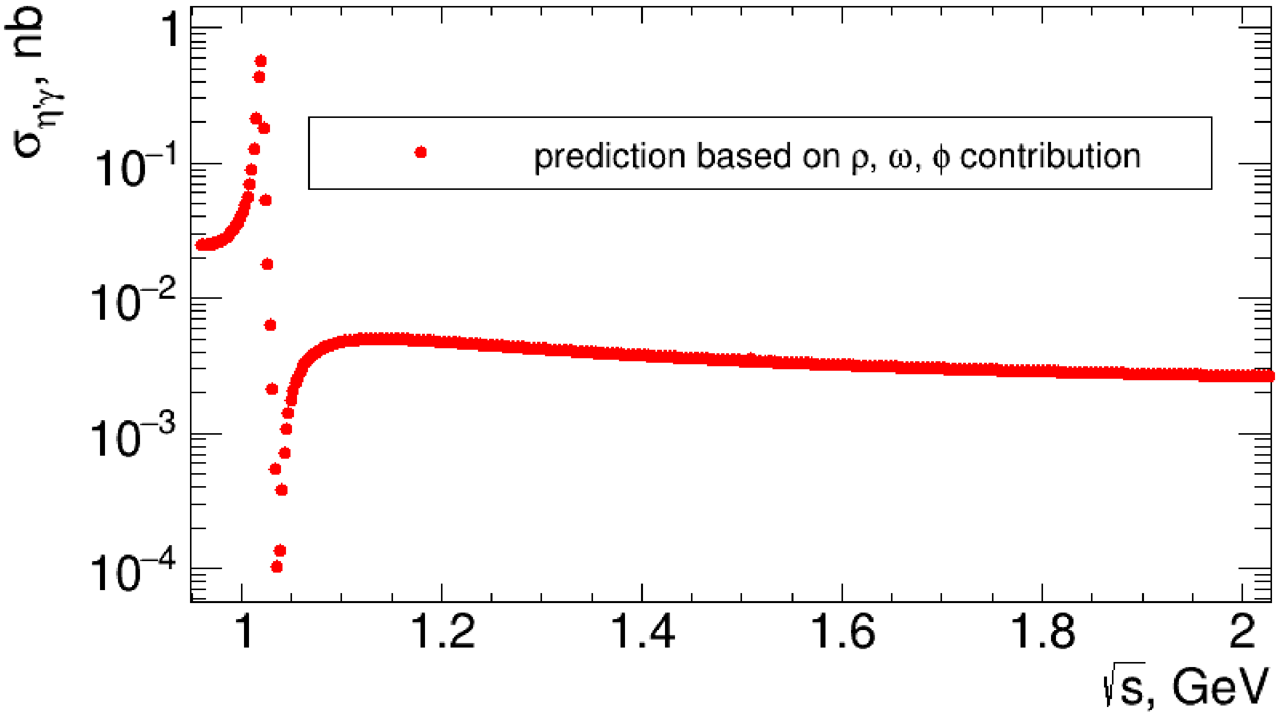
- About 46 events of $e^+e^- \rightarrow e^+e^-\eta'$ were observed in the double tagged mode for the first time.

- The $\gamma^*\gamma^*\rightarrow\eta'$ transition form factor $F(Q^2_1, Q^2_2)$ have been measured for Q^2 range from 2 to 60 GeV^2.

- The form factor is in reasonable agreement with the pQCD prediction.

- We propose a measurement of this quantity at BELLE II.

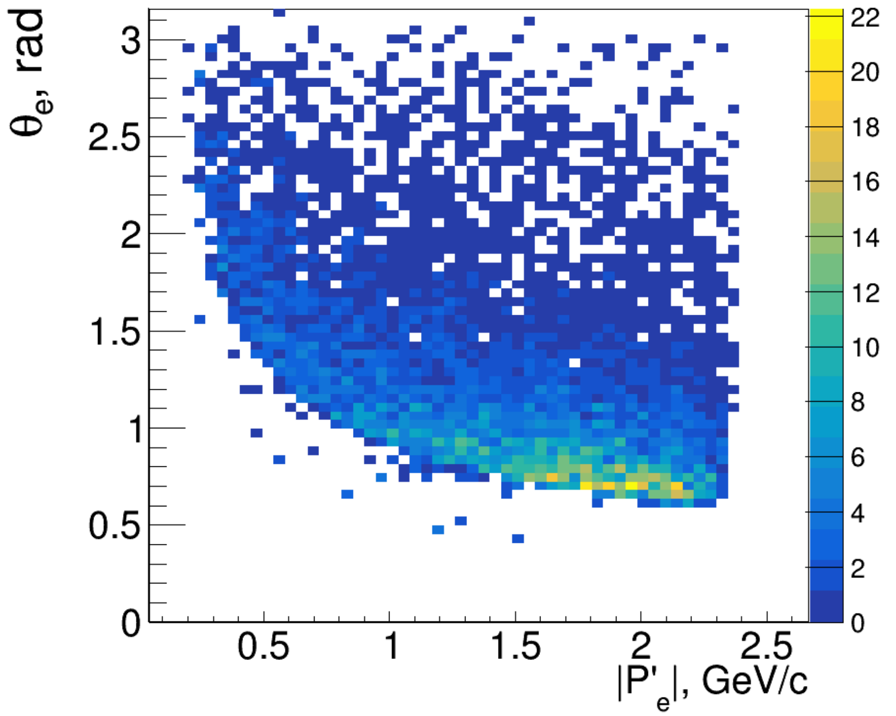
The estimation of $e^+e^- \to \eta`\gamma$ cross section based on the contribution of ρ, ω, φ mesons.



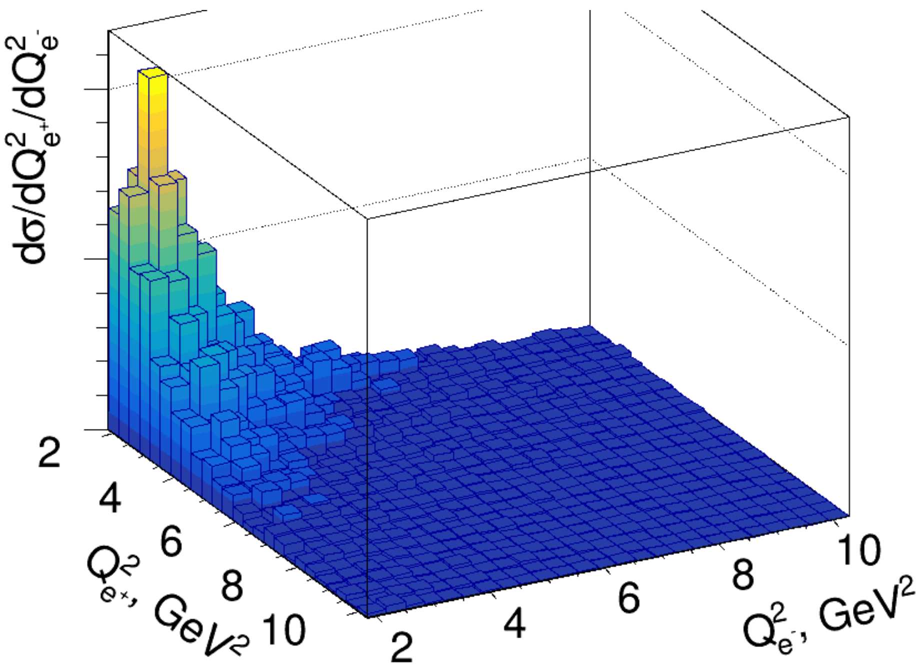
We need 10 pb^{-1}/point with VEPP-2000 at least for measurement of the cross section above φ meson.

Let us consider the e^+e^- collisions at $E_{c.m.} = 5$ GeV.

The obtained TFF allows us to predict $\boldsymbol{\sigma_{e+e-\to e+e-\eta'}(Q_1^2, Q_2^2 > 2\ \mathrm{GeV}^2) = 3.06 +/- 0.01\ \mathrm{fb}}$



The angle vs momentum of scattered fermion



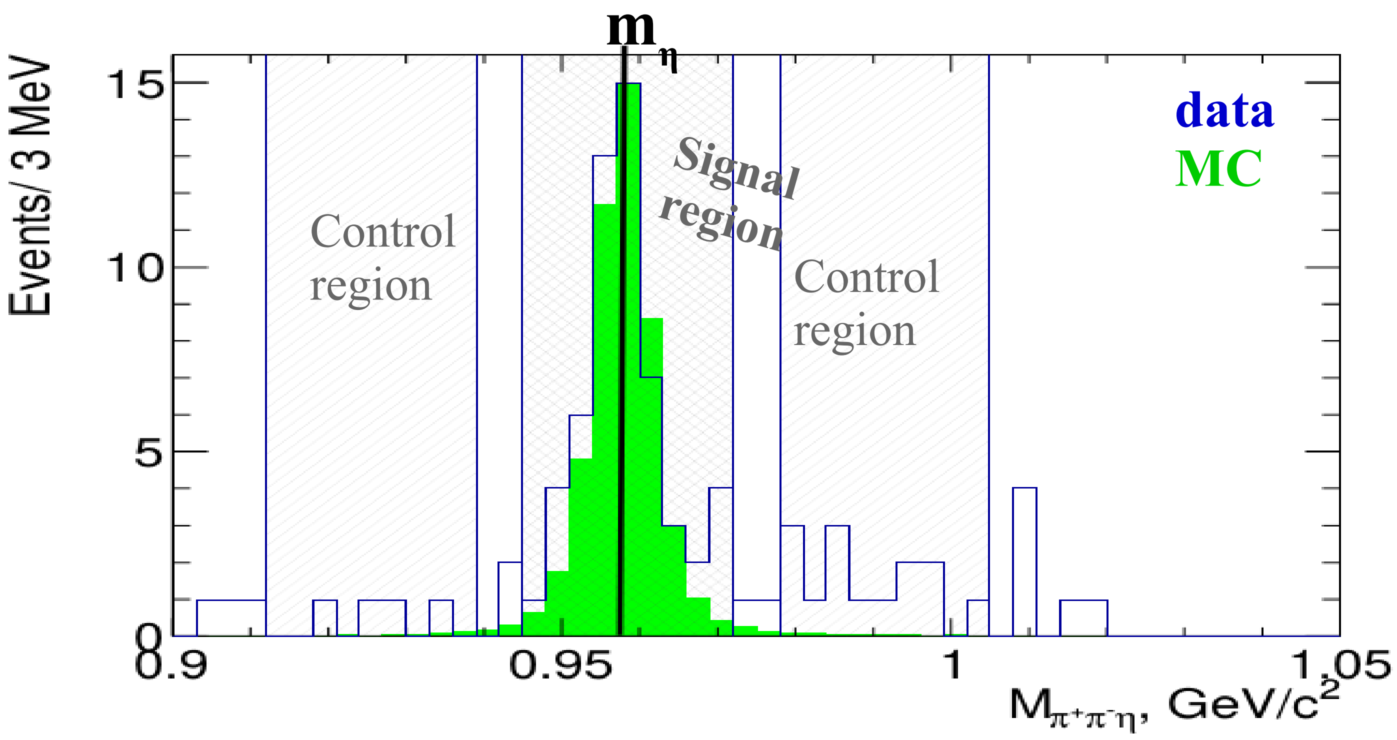
The differential cross section

The measurement of double off-shell TFF is a challenge and can be performed only at experiments with super high luminosity.

Thank you for your attention

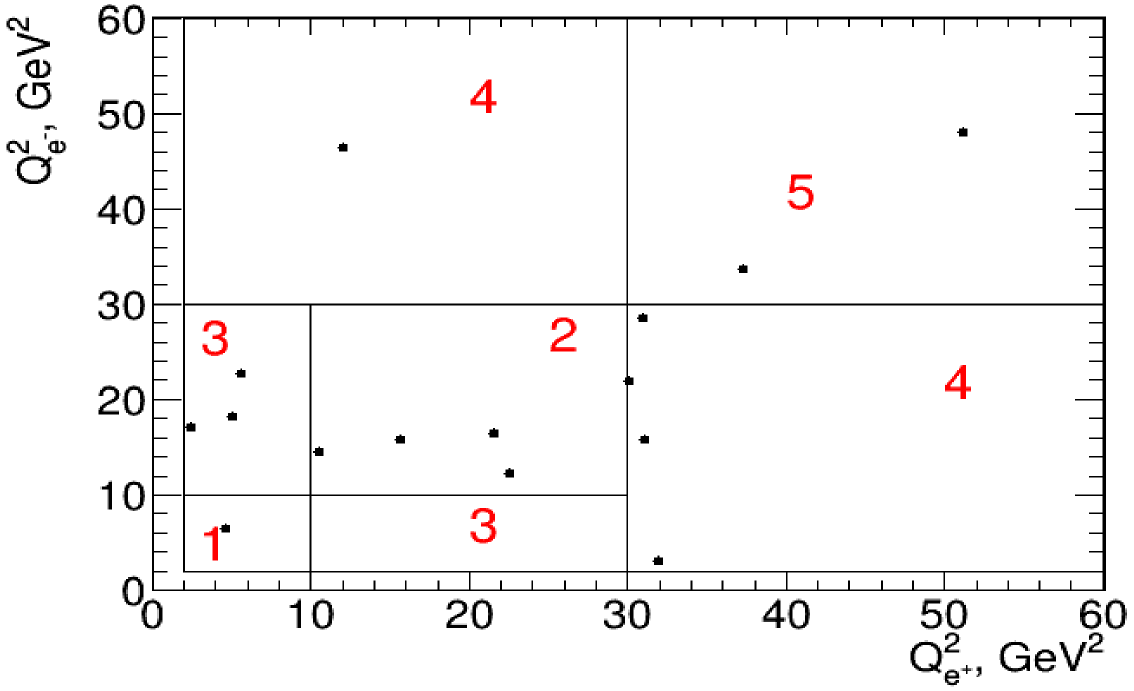
Back up slides

Event selection



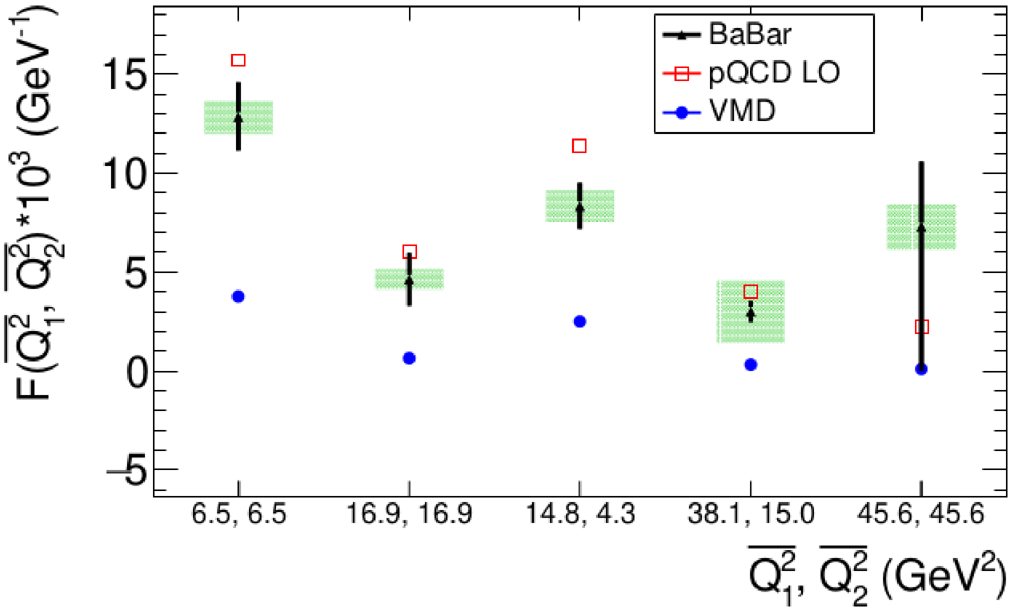
The data-MC comparison of ππη invariant mass distribution. The MC histogram is normalized to central bin of data distribution.

The expected number of signal $\mathbf{N}^{\mathbf{side}}_{\mathbf{signal}} = 55 - 18/2 = 46$



The $Q^2{}_{e-}$ vs. $Q^2{}_{e+}$ for events from control side-band regions

If $(d^2\sigma/(dQ^2_1\, dQ^2_2))_{MC}$ and ε_{true} is made using VMD TFF:



The comparison of obtained form-factor with theoretical predictions. The Error bars - statistical uncertainties. Shaded rectangles - quadratic sum of the systematic and model uncertainties.

$$|\eta'> \;=\; \sin\phi\,|n> + \cos\phi\,|s>$$

$$F_{\eta'} \;=\; \sin\phi\, F_n + \cos\phi\, F_s$$

$$|n> \;=\; \frac{1}{\sqrt{2}}(|\bar{u}u> + |\bar{d}d>)$$

$$|s> \;=\; |\bar{s}s>$$

$$|\eta'> \;=\; sin\phi|n> + cos\phi|s>$$

- $$\lim_{Q^2\to\infty} F_n(Q^2) = \frac{5\sqrt{2}}{3Q^2} f_n; \lim_{Q^2\to\infty} F_s(Q^2) = \frac{2}{3Q^2} f_s$$

Master formula

- $$F_{\eta'}(Q_1^2, Q_2^2) = (\frac{5\sqrt{2}}{9}\cdot f_n \cdot \sin\phi + \frac{2}{9}\cdot f_s\cdot\cos\phi)\cdot \int_0^1 dx \frac{3x(1-x)}{xQ_1^2 + (1-x)Q_2^2}(1 + C_F\frac{Q^2}{2\pi}\cdot t(x, Q_1^2, Q_2^2)) + (x \to 1-x)$$

- at which scale of Q^2 the asymptotic pQCD perdiction starts to be valid?

- In the case of $\gamma\gamma^* \to P$:

$$F_{\eta'}(Q^2) = F_{\eta'}(Q^2, 0) = \frac{\frac{5\sqrt{2}}{9}\cdot f_n\cdot\sin\phi + \frac{2}{9}\cdot f_s\cdot\cos\phi}{Q^2}\cdot\left(1 - \frac{5}{2}C_F\frac{\alpha_S(Q^2)}{2\pi}\right)$$